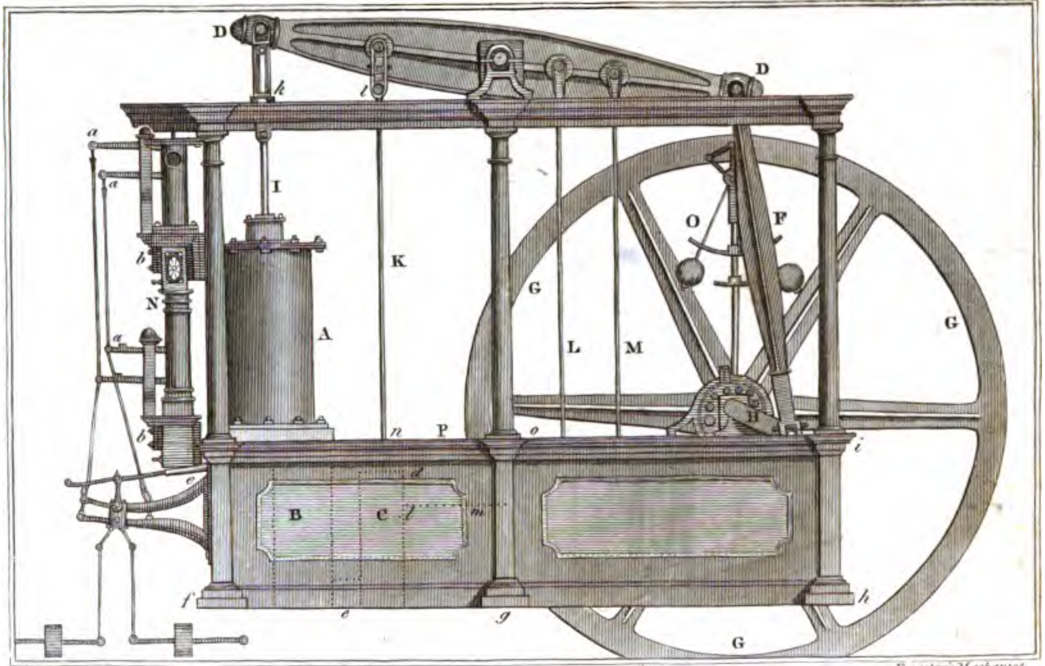

SIGILLVM ACADEMIÆ HARVARDIANÆ IN NOV: ANG:
CHRISTO ET ECCLESIÆ

FRONTISPIECE



Rollinson Sc.

Brunton's Mechanics.

Published by G. & C. & H. Carvill 1830.

A COMPENDIUM OF MECHANICS;

OR

TEXT BOOK

FOR

ENGINEERS, MILL-WRIGHTS, MACHINE-MAKERS, FOUNDERS, SMITHS, &c.

CONTAINING

PRACTICAL RULES AND TABLES

CONNECTED WITH THE

STEAM ENGINE, WATER WHEEL, PUMP, AND MECHANICS IN GENERAL;

ALSO,

EXAMPLES FOR EACH RULE,

CALCULATED IN DECIMAL ARITHMETIC,

Which renders this Treatise particularly adapted for the use of

OPERATIVE MECHANICS.

BY ROBERT BRUNTON.

TO WHICH HAVE BEEN ADDED

VARIOUS TABLES AND RULES FOR CALCULATION,

TOGETHER WITH THE

ELEMENTS OF ISOMETRICAL PERSPECTIVE.

First American from *Fourth London* Edition, *with Plates.*

EDITED BY JAMES RENWICK, LL. D.
Professor of Natural Experimental Philosophy and Chemistry in Columbia College.

NEW-YORK:
G. & C. & H. CARVILL, 108 BROADWAY.
....
1830.

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of Saratoga Springs.

Printed by G. F. Bunce, 224 Cherry-st.

PREFACE.

The following Compilation is submitted to the Mechanics of Glasgow, by one of their number, who hopes it will be found a simple and easy Introduction to the knowledge of Calculation connected with Mechanics.

Most of the Rules and Tables have been selected from the latest eminent Publications on these subjects, and information procured from every possible source, with a view of rendering this Work useful for practical purposes.

The want of a Text Book for Operative Mechanics has been long felt.—The great inconvenience arising from this, was the cause of the Compiler collecting the following Rules for his own personal use:—and having, with several other Mechanics, experienced the great advantage derived from these Memoranda, he is induced to submit them to the Public, trusting they will be found to contain much useful information.

GLASGOW,
February, 1824.

PREFACE

TO THE

FOURTH EDITION.

THE increasing demand for this little Work, is a proof that it is found useful; and the Compiler wishing to render it still more so, has added a few Geometrical Problems and Tables of Roots, Squares, and Cubes, which he hopes will generally be found serviceable.

It would be an easy task to swell this Book to a much larger size, by adding Problems and other matter, which might be considered by some useful; but it was originally intended for Operative Mechanics, and the Compiler has no wish to alter the plan; to them he is indebted for the favours he has received, and their advantage he intends to study; the additions made must, therefore, be of practical utility.

It is highly gratifying to trace the rapid advance of knowledge among Mechanics, and the many excellent opportunities they now have for its attainment:—that much-wished-for time appears to be at hand, when Mechanics shall not only be acknowledged cunning artificers, but men of science:—when the word Mechanic shall convey the idea of wisdom and understanding,—and the profession, highly fraught with good to man, shall be honoured and respected.

LONDON.
July, 1828.

EXPLANATIONS

OF THE

CHARACTERS USED IN THE FOLLOWING CALCULATIONS.

$+$ Signifies Addition, as $5 + 3$ is 8.
$-$ ——— Subtraction, as $5 - 3$ is 2.
$\times$ ——— Multiplication, as 5×3 is 15.
$\div$ ——— Division, as $15 \div 3$ is 5, or $\frac{15}{3}$ is 5.
$: :: :$ ——— Proportion, as 2 is to 3, as 4 is to 6.
$=$ ——— Equality, as $5 + 3 = 8$.
$\surd$ ——— Square Root, $\sqrt{9} = 3$.
$\sqrt[3]{\ }$ ——— Cube Root, $\sqrt[3]{27} = 3$.

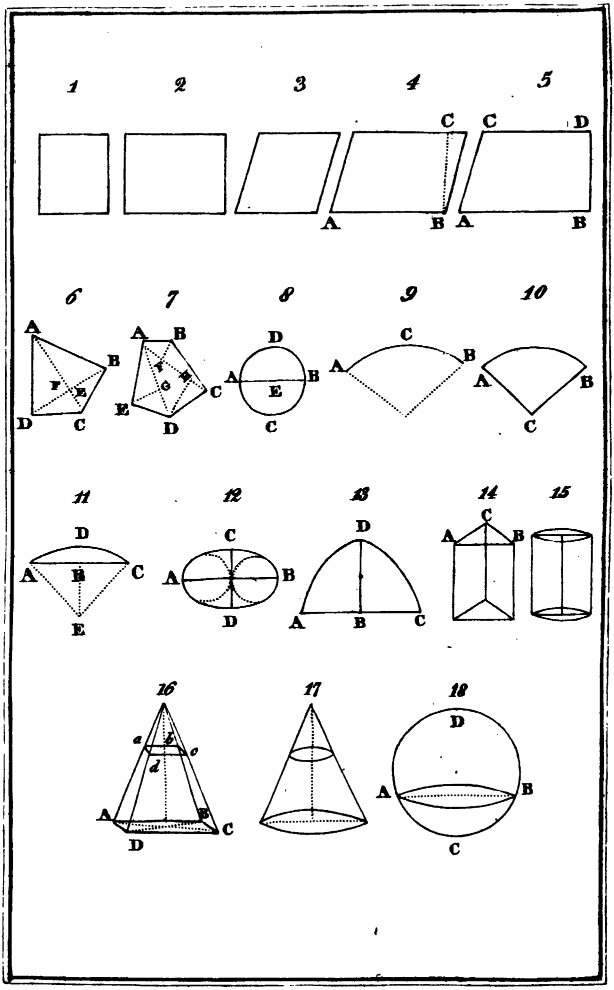
3^2 Signifies that 3 is to be squared as $3^2 = 9$.
3^3 ——— 3 is to be cubed as $3^3 = 27$.
The Bar signifies that 2 numbers are to be taken together, as $3 \times \overline{5 + 3} = 24$.

DESCRIPTION OF FRONTISPIECE.

A Cylinder.
B Condenser.
C Air Pump.
D D Working Beam.
F Connecting Rod, or Shackle Bar.
G G G Fly Wheel.
H Crank.
I Piston Rod,
K Air Pump Rod.
L Hot Water Pump Rod.
M Cold Water Pump Rod.
N Side Pipes.
O Governor.
P Hot Water Cistern.
a a a a Hand Gear for Valves.
b b Valve Seats.
c Foot Valve.
d Delivering Door.
e f g h i Cold Water Cistern.
k l Parallel Motion.

MENSURATION.

Plate 1st



EXPLANATION OF PLATE FIRST.

No. 1. A Square.

2. A Rectangle.

3. A Rhombus.

4. A Rhomboid.

5. A Trapezoid.

6. A Trapezium.

7. An Irregular Polygon.

8. A Circle, A E the radius, A E B the diameter, A C B D the circumference.

9. An Arc of a Circle, A C B.

10. A Sector of a Circle, A C B.

11. A Segment of a Circle, A B C D.

12. An Ellipsis or Oval, A B the long diameter, C D the short diameter.

13. A Parabola, A B C the base, B D the perpendicular height.

14. A Prism, A B C the perimeter; or the circumference of the end of a cylinder, is the perimeter of that cylinder.

15. A Cylinder.

16. A Pyramid, A B C D the base; A B C D *a b c d* the frustum.

17. A Cone.

18. A Sphere, A B C D the circumference, A B C a segment.

EXPLANATION OF PLATE SECOND.

A—Lever of the first order.

B—Lever of the second order.

C—Lever of the third order.

D—Bended Lever: the effective power and weight on a bended lever, is as the distance between the points of action and the fulcrum, as *a b c b*. The distance being taken at right angles to the direction of the forces.

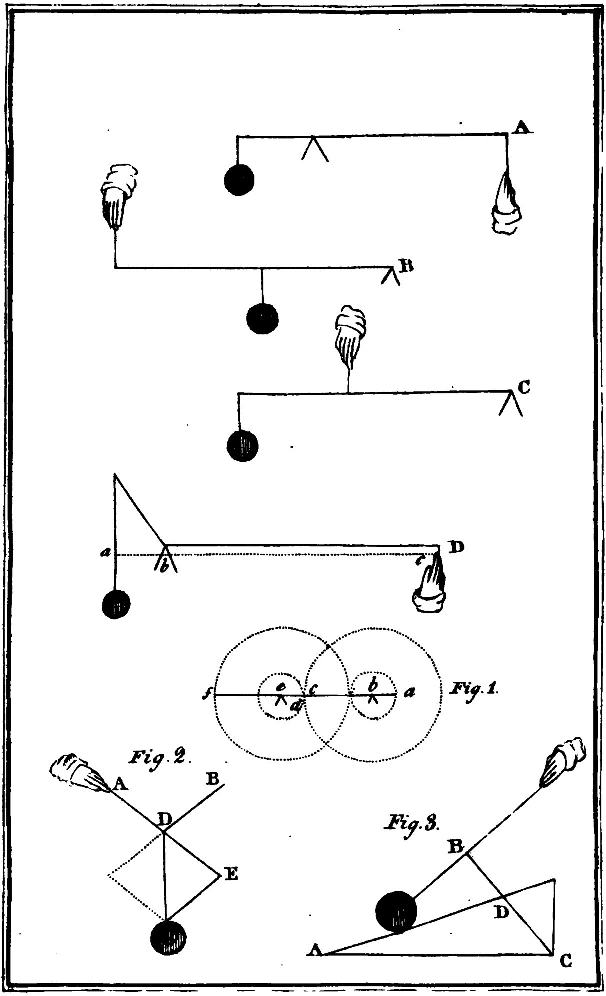
Fig. 1. A Diagram, explanatory of the Wheel and Axle.

Fig. 2. A Diagram, explanatory of the Pulley, when the directions of the cords are not parallel.

Fig. 3. A Diagram, explanatory of the inclined plane, when the power is not in a direction with the plane.

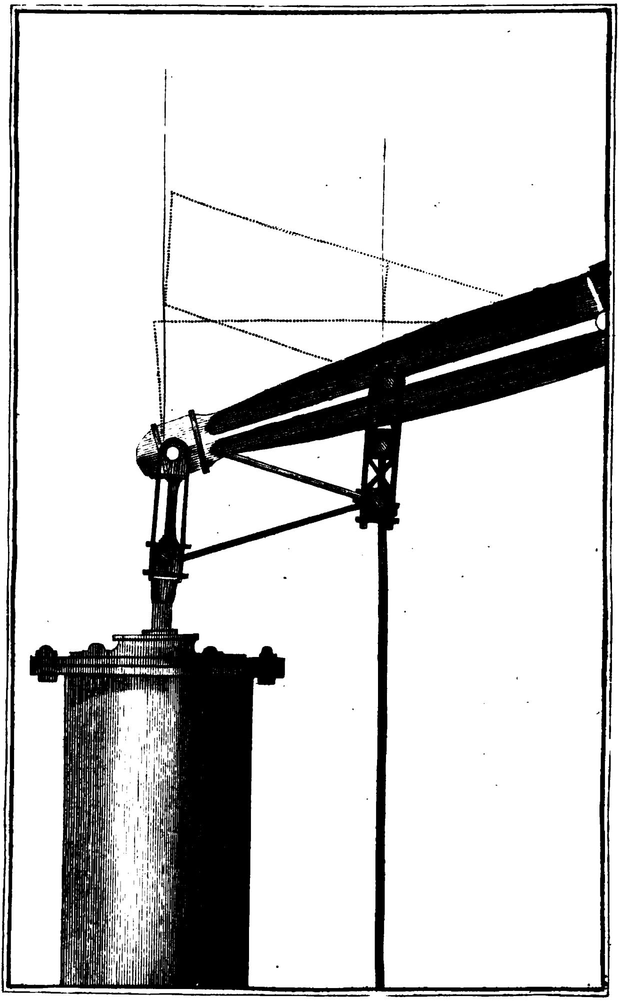
Mechanical Powers.

Plate 2d



Parallel Motion

Plate 3d.



DIAGRAMS.

Plate 4.

Fig. 1.

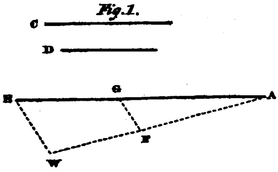


Fig. 2.

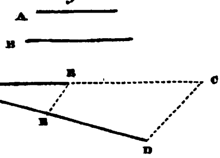


Fig. 3.

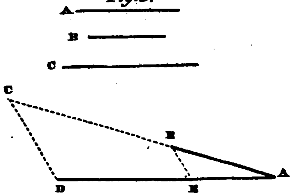


Fig. 4.

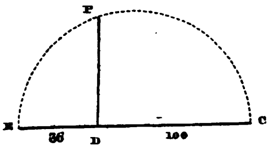


Fig. 5.

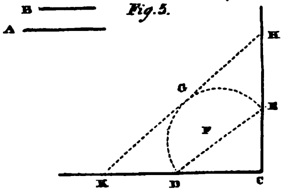


Fig. 6.

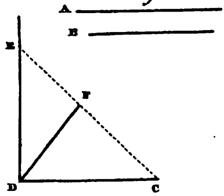


Fig. 7.

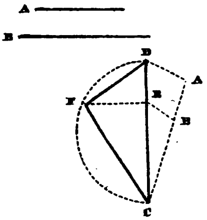
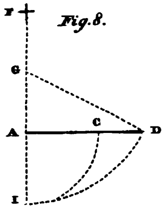


Fig. 8.



The rood of land shall contain 1210 square yards, an acre 4840 square yards, or 160 square perches, poles or rods.

STANDARD POUND.

A cubic inch of distilled water, weighed in air by brass weights, at the temperature of 62 degrees Fahrenheit, the barometer being at 30 inches, is equal to 252 grains and 458 thousandth parts of a grain, of which the Standard Troy Pound shall contain 5760.

STANDARD GALLON.

The Standard Gallon shall contain 10 Pounds Avoirdupois weight of distilled water weighed in air, at the temperature of 62° Fahrenheit, the barometer being at 30 inches.

STANDARD FOR HEAPED MEASURE.

The Standard for heaped Measure shall contain 80 pounds Avoirdupois weight, with a plain and even bottom, and 19½ inches from outside to outside, of such standard measure as aforesaid.

STANDARD WEIGHTS AND MEASURES OF THE STATE OF NEW-YORK.

By the Revised Laws of the State of New-York, a system of Weights and Measures has been adopted; the principles of which are as follows, viz:—

STANDARD YARD.

The Standard Yard of the State of New-York bears to a Pendulum vibrating seconds in a vacuum

in Columbia College, the relation of 1,000,000 to 1,086,141. Its Standard temperature is 32°—that of the English 62°.

STANDARD POUND.

Of the two denominations of weight, the Troy and Avoirdupois, the latter is alone retained; and its unit is defined by declaring that a cubic foot of pure water at its maximum density weighs 62½ such pounds, or 1,000 ounces, using brass weights, at the mean pressure of the atmosphere at the level of the sea.

STANDARD OF DRY MEASURE.

The gallon of Dry Measure shall contain ten pounds of pure water, at its maximum density; the bushel eighty pounds, for articles sold by heaped measure: the vessel that measures the bushel shall be cylindrical, and 19½ inches in diameter from outside to outside.

The half bushel, ... 15½ in.
The peck, ... 12⅓ in.

STANDARD FOR LIQUID MEASURE.

For reasons similar to those which influenced the British Parliament, the revised laws of the State of New-York originally provided but one standard for measures of capacity, both dry and liquid; but, as from a misapprehension of former laws, the wine gallon had become the usual measure for liquids, the Legislature have admitted it as a standard for all liquid measures, and enacted that it shall contain 8 lbs. of pure water at its maximum of density.

	Cub. in.
The bushel therefore measures . .	2211.84
The gallon of Dry measure . . .	276.48
The gallon of Liquid measure . .	221.18

These measures have this great advantage, in ordinary use, that common pump or spring water, fresh drawn, is sufficiently near the standard density to be employed for regulating them, in all cases where scientific accuracy is not required.

A. E.

TROY WEIGHT.

Gr.	*Dwt.*		
24 =	1	*Oz.*	
480 =	20 =	1	*Lib.*
5760 =	240 =	12 =	1

	Mark.
Grains, ...	*Gr.*
Penny-weights, .	*Dwt.*
Ounce, ...	*Oz.*
Pound, ...	*Lib.*

By this weight are weighed Gold, Silver, and Jewels.

AVOIRDUPOIS WEIGHT.

Dr.	*Oz.*				
16=	1	*Lib.*			
256=	16=	1	*Qr.*		
7168=	448=	28=	1	*Cwt.*	
28672=	1792=	112=	4=	1	*Ton.*
573440=	35840=	2240=	80=	20=	1

	Mark.
Dram, . .	*Dr.*
Ounce, .	*Oz.*
Pound, .	*Lib.*
Quarter, .	*Qr.*
Hund. wt.	*Cwt.*
Ton, . .	*Ton.*

By this Weight all Metals, except Gold and Silver, are weighed.

In the United States the denominations of Avoirdupois weight greater than the pound are rarely used. In some of the States they are forbidden by law.

		Oz.	Dwt.	Gr.	
Note. 1 Lib. Avoir.	=	14 .	11 .	$15\frac{1}{2}$	Troy.
1 Oz. do.	=		18 .	$5\frac{1}{2}$	do.
1 Dr. do.	=		1 .	$3\frac{1}{2}$	do.

LONG MEASURE.

							Mark.
In.	*Ft.*						Inch,……*In.*
12=	1	*Yd.*					Foot,……*Ft.*
36=	3 =	1	*Fath.*				Yard,……*Yd.*
72=	6 =	2 =	1	*Pl.*			Fathom,..*Fath.*
198=	$16\frac{1}{2}$ =	$5\frac{1}{2}$=	$2\frac{3}{4}$=	1	*Fur.*		Pole or Rod, *Pl.*
7920=	660 =	220 =	110 =	40=	1	*M.*	Furlong,…*Fur*
63360=	5280 =	1760 =	880 =	320=	8=	1	Mile,……*M.*

3 Miles = 1 League, marked Lea.
$2\frac{3}{4}$ do. = 1 French League.
$3\frac{2}{3}$ do. = 1 Spanish League.
4 do. = 1 German Mile.
$3\frac{1}{4}$ do. = 1 Dutch Mile.
$1\frac{1}{2}$ do. = 1 Italian Mile.
$\frac{3}{4}$ do. = 1 Russian Verst.
$1\frac{3}{22}$ do. = 1 Scotch Mile.
$1\frac{3}{11}$ do. = 1 Irish Mile.
$69\frac{1}{15}$ Miles nearly = 1 Degree, marked °

SQUARE MEASURE.

						Mark.
Sq. In.	*Sq. Ft.*					Square Inch, *Sq. In.*
144=	1	*Sq. Yd.*				——Foot, *Sq. Ft.*
1296=	9 =	1	*Sq. Pl.*			——Yard, *Sq. Yd.*
39204=	$272\frac{1}{4}$=	$30\frac{1}{4}$=	1	*Rd.*		——Pole, *Sq. Pl.*
1568160=	10890 =	1210 =	40=	1	*Acr.*	Rood,……*Rd.*
6272640=	43560 =	4840 =	160=	4=	1	Acre,……*Acr.*

1089 Scotch Acres = 1369 English Acres.

DRY MEASURE.

							Mark.
Pts.	*Gal.*					Pints,	*Pts.*
8=	1	*Pec.*				Gallon,	*Gal.*
16=	2=	1	*Bu.*			Peck,	*Pec.*
64=	8=	4=	1	*Qr.*		Bushel,	*Bu.*

The denominations of this measure greater than the bushel, are not used in the United States.

512=	64=	32=	8=	1 *Wey*			Quarter,	*Qr.*
2560=	320=	160=	40=	5=	1 *Last*		Wey, Load, or Ton,	*Wey.*
5120=	640=	320=	80=	10=	2=	1	Last,	*Last.*

A Chaldron of Coals in London = 36 Bushels, and weighs 3136 libs Avoirdupois, or 1 Ton 8 Cwt. nearly.

The same Chaldron is used in the City of New-York.

ALE MEASURE.

								Mark.
Pts.	*Qt.*						Pints,	*Pts.*
2=	1	*Gal.*					Quart,	*Qt.*
8=	4=	1	*Bar.*				Gallon,	*Gal.*
288=	144=	36=	1	*Hhd.*			Barrel,	*Bar.*
432=	216=	54=	$1\frac{1}{2}$=	1	*Butt*		Hogshead,	*Hhd.*
864=	432=	108=	3 =	2=	1	*Tun*	Butt,	*Butt.*
1728=	864=	216=	6 =	4=	2=	1	Tun,	*Tun.*

Note. The Ale Gallon contains 282 Cubic or Solid Inches.

Ale Measure is not used in the State of New-York.

WINE MEASURE.

(*Liquid Measure of the State of New-York.*)

				Mark.
Pts.	*Qt.*		Pints,	*Pts.*
2=	1	*Gal.*	Quart,	*Qt.*
8=	4=	1 *Tier.*	Gallon,	*Gal.*

This measure is used in the Customs of the United States.

336=	168=	42=	1	*Hhd.*				Tierce,	*Tier.*
504=	252=	63=	$1\frac{1}{2}$=	1	*Pun.*			Hogshead,	*Hhd.*
672=	336=	84=	2 =	$1\frac{1}{2}$=	1	*Pipe*		Puncheon,	*Pun.*
1008=	504=	126=	3 =	2 =	$1\frac{1}{2}$=	1	*Tun*	Pipe or Butt,	*Pipe*
2016=	1008=	252=	6 =	4 =	3 =	2=	1	Tun,	*Tun.*

Note. The Wine Gallon of the Old English standard contained 231 Cubic or Solid Inches; and it is remarkable, that the Wine Gallon was to the Ale Gallon, nearly as the Pound Troy is to the Pound Avoirdupois.

The Liquid Gallon of the State of New-York contains 221.18 Cubic Inches. The Barrel for Ale and Beer 32 such gallons.

IMPERIAL STANDARD GALLON

Contains 10 libs Avoirdupois, and its cubic contents 277.2738 inches.

The above for all sorts of liquids; as for dry goods not measured by heap measure, shall be computed; and all Measures shall be taken in certain proportions of the said Imperial Standard Gallon: the Quart, the fourth part thereof; the Pint, one-eighth thereof—two such Gallons shall be a Peck—eight such Gallons shall be a Bushel—eight such Bushels shall be a quarter of corn or other dry goods—not heaped measure.

The New-York Standard Gallon for Dry Measure contains 276.48 Cubic Inches.

SOLID MEASURE.

Cubic Inches.		*Cub. Ft.*				
1728	=	1		*Cub. Yd.*		
15552	=	9	=	1		*Fathom.*
373248	=	216	=	8	=	1

In taking the solid contents of any Mass, there is seldom any other Measure than the Cubic Foot used.

*The Old and New French Weights and Measures, reduced to the English Standard.**

The Paris pound, *poids de marc of Charlemagne*, contains 9216 Paris grains; it is divided into 16 ounces, each ounce into 8 gros, (or drams,) and each gros into 72 grains; it is equal to 7561 English troy grains.

The English troy pound of 12 ounces, contains 5760 English troy grains, and is equal to 7021 Paris grains.

The English avoirdupois pound of 16 ounces, contains 7000 English troy grains, and is equal to 8538 Paris grains.

To reduce Paris grains to English troy grains, divide by 1.2189.

To reduce Paris ounces to English troy, divide by 1.015734; or the conversion may be made by means of the following Tables.

I. To reduce French to English Troy Weight.

		English Troy Grains.
The Paris Pound	=	7561
Ounce	=	472.5625
Gros	=	59.0703
Grain	=	.8204

* See the Technical Repository, Vol. 3, No. 6, for June 1823.

II. To reduce Paris Long Measure to English.

		Eng. Inches.
The Paris Royal Foot of 12 Inches	=	12.7977
The Inch	=	1.0659
The Line, or one twelfth of an Inch	=	.0074

III. To reduce French Cubic Measure to English.

		Eng. Cubical Feet.
The Paris Cubic Foot . .	=	1.211273
The Cubic Inch	=	.000700

IV. Measure of Capacity.

The Paris pint contains 58.145 English cubical inches, and the English Wine pint contains 28.875 cubical inches; or the Paris pint contains 2.0171082 English pints; therefore, to reduce the Paris pint to the English, multiply by 2.0171082.

TABLE *of the New French Weights and Measures reduced to the English Standard.*

The French *metre*, according to the *Journal de Physique*, *An.* 7. *Prair*, *&* *Fruct*, is equal to 3 feet, 11.296 lines French, and the *gramme* to 18.827 grains. The *metre* is the ten-millionth part of the distance from the Pole to the Equator. The *gramme* is the weight of a cubic centimetre of water. The French *toise* was 76.734 inches English; and 576 French grains were equal to 472.5 English.——*See Phil. Transact.* vol. 58, p. 326.

MEASURES OF LENGTH.

		English Inches.
Millimetre	= . . .	.03937
Centimetre	=	.39370
Decimetre	= . . .	3.93702
Metre	= . . .	39.37023
Decametre	= .	393.70226
Hecatometre	= .	3937.02260
Chiliometre	=	39370.22601
Myriometre	=	393702.26014

		M.	P.	Y.	Ft.	In.
A Decametre is	=	0	0	10	2	9.7
A Hecatometre	=	0	0	109	1	.1
A Chiliometre	=	0	4	213	1	10.2
A Myriometre	=	6	1	156	0	.6

Eight Chiliometres are nearly 5 English miles.

MEASURES OF CAPACITY.

		English Cubic Inches.
Millilitre	=	.06102
Centilitre	=	.61024
Decilitre	=	6.10244
Litre	= . . .	61.10244
Decalitre	= . . .	610.24429
Hecatolitre	= . .	6102.44288
Chiliolitre	= .	61024.42878
Myriolitre	= .	610244.28778

A Litre is nearly 2⅛ Wine pints.
14 Decilitres are nearly 3 Wine pints.
A Chiliolitre is a tun, 12.75 Wine gallons.

WEIGHTS.

		English Grains.
Milligramme	= . .	.0154
Centigramme	=	.1544
Decigramme	= . . .	1.5444
Gramme	= . . .	15.4440
Decagramme	= . .	154.4402
Hecatogramme	=	1544.4023
Chiliogramme (Kilogram)	= .	15444.0234
Myeiogramme . . .	= .	154440.2344

A Decagramme is 6 dwts. 10.44 gr. tr.; or 5.65 dr. avoir.

A Hecatogramme is 3 oz. 8.5 dr. avoir.

A Chiliogramme is 2 libs. 3 oz. 5 dr. avoir.

A Myriogramme is 22 — 1.15 oz. avoir.

100 Myriogrammes are 1 ton, wanting 32.8 libs.

AGRARIAN MEASURES.

Are, 1 square Decametre .	= 3.95 Perches.
Hecatare	= 2 Acres, 1 Rood, 30.1 Perches.

FIR WOOD.

Decistre, 1-10th Stere .	= 3.5315 cub. ft. Eng.
Stere, 1 Cubic Metre .	= 35.3150 cub. ft.

MENSURATION.

AREAS OR SURFACES.

PROBLEM I.

To find the area of any Parallelogram, wheether it be a Square, a Rectangle, a Rhombus, or a Rhomboid. See Plate I. Fig. 1, 2, 3, 4.

Rule. Multiply the length by the perpendicular breadth or height, and the product will be the area.

EXAMPLE.

What is the area of a Rhomboid, the length A B being 7, the perpendicular B C being 4?

$$7 \times 4 = 28 \text{ area of Rhomboid.}$$

PROBLEM II.

To find the area of a Triangle.

Rule. Multiply the base by the perpendicular height, and take half the product for the area.

EXAMPLE.

What is the area of a Triangle, the base being 9, and the perpendicular $8\frac{1}{2}$?

$$9 \times 8\tfrac{1}{2} = 76\tfrac{1}{2} \frac{76\frac{1}{2}}{2} = 38\tfrac{1}{4} \text{ area.}$$

PROBLEM III.

To find the area of a Trapezoid.—See plate I. Fig. 5.

RULE. Add together the two parallel sides; then multiply their sum by the perpendicular breadth, or the distance between them, and take half the product for the area.

EXAMPLE.

What is the area of a Trapezoid?

A B = 9, C D = 8 } parallel sides.	$\overline{9+8} \times 5 = 85$
B D = 5 perpendicular.	and $\frac{85}{2} = 42\frac{1}{2}$ area.

PROBLEM IV.

To find the area of any Trapezium.—See Fig. 6.

RULE. Divide the Trapezium into two triangles by a diagonal; then fiind the areas of the triangles by Prob. 2. and add them together for the area of the Trapezium.

EXAMPLE.

What is the area of a Trapezium?

A C = 8 base of both triangles.	$\overline{5+4} \times 8 = 72$
D F = 4, B E = 5 } perpendiculars.	and $\frac{72}{2} = 36$ area.

PROBLEM V.

To find the area of an Irregular Polygon.—See Fig. 7.

RULE. Draw Diagonals dividing the proposed Polygon into Trapeziums and Triangles; then find the areas of all these separately, and add them together for the contents of the whole Polygon.

EXAMPLE.

What is the area of an irregular Polygon?

$\left.\begin{array}{l} AC = 9 \\ AD = 8 \end{array}\right\}$ bases.	$\overline{4+2} \times 9 = 54$
	$3 \times 8 = 24$
$\left.\begin{array}{l} BF = 2 \\ DH = 4 \\ EG = 3 \end{array}\right\}$ perpendiculars.	$\overline{78}$
	$\frac{78}{2} = 39$ area.

PROBLEM VI.

To find the area of a Regular Polygon.

RULE 1. Multiply the perimeter of the Polygon, or sum of its sides, by the perpendicula. drawn from its centre on one of its sides, and take half the product for the area.

RULE. 2. Square the side of the Polygon; then multiply that square by the tabular area set against its name in the following Table, and the product will be the area.

EXAMPLE.

What is the area of a regular Nonagon, its side being 5, and perpendicular 6.8686935?

By Rule 1.

45 Perimeter = 5 × 9

45 × 6.8686935 = 309.0912097

now $\frac{309.09}{2}$ &c. = 154.54 &c. area.

By Rule 2.

5^2 × 6.1818242 = 154.5456 &c. area.

No. of sides.	NAMES.	AREAS.
3	Trigon, or Triangle . .	0.4330127
4	Tetragon, or Square .	1.0000000
5	Pentagon	1.7204774
6	Hexagon	2.5980762
7	Heptagon	3.6339124
8	Octagon	4.8284271
9	Nonagon	6.1818242
10	Decagon	7.6942088
11	Undecagon	9.3656399
12	Dodecagon	11.1961524

PROBLEM VII.

To find the diameter and circumference of any Circle, the one from the other.—See Fig. 8.

This may be done by either of the three following proportions, viz. As 7 is to 22, so is the diameter to the circumference; or, As 1 is to 3.1416, so is

the diameter to the circumference; or, As 113 is to 355, so is the diameter to the circumference.

EXAMPLE.

What is the circumference of a Circle, its diameter being 6?

First, $7 : 22 :: 6 : 18.857$ circumference.

Second, $1 : 3.1416 :: 6 : 18.8496$ do.

Third, $113 : 355 :: 6 : 18.8495$ do.

This last (113 : 355) is the most correct proportion to find the circumference from the diameter, or the diameter from the circumference.

PROBLEM VIII.

To find the length of any arc of a Circle.—See Fig. 9.

RULE. Multiply the decimal .01745 by the degrees in the given arc, and the product by the radius of the circle, for the length of the arc.

EXAMPLE.

What is the length of the arc of a Circle, the number of degrees being 25, and radius 7?

$.01745 \times 25 \times 7 = 3.05375$, length of arc.

Note. .01745 is found by dividing the circumference by 360° when the radius is 1 —— *i. e.*
$\frac{6.2831854}{360} = .01745$

PROBLEM IX.

To find the area of a Circle.

RULE 1. Multiply half the circumference by half the diameter, and the product is the area.

RULE 2. Square the diameter, and multiply that square by the decimal .7854 for the area.

RULE 3. Square the circumference and multiply that square by the decimal .07958.

EXAMPLE.

What is the area of a Circle, the diameter being 9, the circumference = 28.27, half of which is 14.135?

By Rule 1.	*By Rule* 2.
$14.135 \times 4.5 = 63.6075$	$9^2 \times .7854 = 63.6174$

By Rule 3.

$28.27^2 \times .07958 = 63.599$, &c.

Note. The first Rule of this Problem is found by the theorem of the triangle: for suppose the circle to be a regular Polygon of an indefinite number of sides, then, the sum of the sides will be the perimeter of the circle; consequently, the radius of the circle will be the altitude, and the perimeter the base of the triangle, the area of which is found by A $\times$ ½ B, or ½ A $\times$ B, (A being the altitude and B the base,) therefore the area of the circle will be ½ cir. $\times$ ½ diameter, or ½ cir. $\times$ radius.

The second Rule is deduced from the first and Prob. 7.: the first Rule is $\frac{DC}{4}$ = area (D C being the diameter and circumference) Prob. 7. is 3.1416 $D = C$, therefore the area is $\frac{3.1416\,D^2}{4}$ or $= .7854\,D^2$

The third Rule is found thus: $D = \frac{C}{3.1416}$ and $A = \frac{DC}{4}$, therefore the area will be $\frac{C^2}{3.1416 \times 4}$ or $\frac{C^2}{12.5664}$ now the reciprocal of 12.5664 is .07958, or $\frac{A}{C^2} =$ .07958; hence the rule $C^2 \times .07958$ = area.

PROBLEM X.

To find the area of a circular Ring, or of the space included between the circumferences of two circles; the one being contained within the other.

Rule. Take the difference between the areas of the two circles for the area of the Ring.

EXAMPLE.

What is the area of a Ring, the inside diameter being 5, and outside 7?

$.7854 \times 7^2 = 38.4846$

$.7854 \times 5^2 = 19.6350$

18.8496 = area of ring, or $\frac{7+5}{2} = 6$ medium diamr. then $3.1416 \times 6 \times \frac{7-5}{2} = 18.84$ area, or $7^2 - 5^2 = 24$, then $.7854 \times 24 = 18.84$ area.

PROBLEM XI.

To find the area of a Sector of a Circle.—See Fig. 10.

RULE. 1. Multiply the radius, or half the diameter, by half the arc of the sector, for the area; or multiply the whole diameter by the whole arc of the sector, and take $\frac{1}{4}$ of the product.

RULE 2. Compute the area of the whole circle: then say, as 360° is to the degrees in the arc of the sector, so is the area of the whole circle, to the area of the sector.

EXAMPLE.

What is the area of the sector of a Circle, the radius A C being 5, and arc A B, 8?

Rule 1. $4 \times 5 = 20$ area, or $\frac{8 \times 10}{4} = 20$ area.

Rule 2. 78.5400 = area of circle, then 360°: 92° 18′ :: 78.54: 20 area, (92° 18′ is the portion of the circle contained in arc 8.)

PROBLEM XII.

To find the area of a Segment of a Circle.—See Fig. 11.

RULE 1. Find the area of the sector, having the same arc with the segment, by the 2nd rule of last Problem. Find also the area of the triangle, formed by the chord of the segment and the two radii of

the sector; then add these together for the answer, when the segment is greater than a semicircle; or subtract them, when it is less than a semicircle; As is evident.

RULE 2. Divide the height of the segment by the diameter, and find the quotient in the column of heights in the following Table. Take out the corresponding area in the next column on the right hand, and multiply it by the square of the circle's diameter for the area of the segment.

When the quotient is not found exactly in the Table, proportion may be made between the next less and greater area, in the same manner as is done with any other Table.

EXAMPLE.

What is the area of the segment of a Circle, the radius A E being 10, chord A B C, 12, and arc A D C 73° 74′?

Rule 1. $.7854 \times 400 = 314.16$ area of whole circle, and 360° : 73° 74′ :: 314.16 : 6434054 area of sector. Half the chord is 6, and $\sqrt{10^2 - 6^2} = 8. =$ B E the height of the triangle.

Rule 2. $10 - 8 = 2$ height of the segment. $\frac{2}{20} = .1$ the quotient; and opposite to it, in the right column of the following Table, is .04088, which multiplied by the square of the diameter, is $.04088 \times 20^2 = 16.352$ area, nearly same area as found by Rule 1.

Table of the area of circular Segments.

Height.	Area of Segment.	Height.	Area of Segment.	Height.	Area of Segment.	Height.	Area of Segment.	Height.	Area of Segment.
.01	.00133	.11	.04701	.21	.11990	.31	.20738	.41	.30319
.02	.00375	.12	.05339	.22	.12811	.32	.21667	.42	.31304
.03	.00687	.13	.06000	.23	.13646	.33	.22603	.43	.32293
.04	.01054	.14	.06683	.24	.14494	.34	.23547	.44	.33284
.05	.01468	.15	.07387	.25	.15354	.35	.24498	.45	.34278
.06	.01924	.16	.08111	.26	.16226	.36	.25455	.46	.35274
.07	.02417	.17	.08853	.27	.17109	.37	.26418	.47	.36272
.08	.02944	.18	.09613	.28	.18002	.38	.27386	.48	.37270
.09	.03502	.19	.10390	.29	.18905	.39	.28359	.49	.38270
.10	.04088	.20	.11182	.30	.19817	.40	.29337	.50	.39270

PROBLEM XIII.

To measure long irregular Figures.

Rule. Take or measure the breadth at both ends, and at several places, at equal distances; then add together all these intermediate breadths, and half the two extremes; which sum multiply by the length, and divide by the number of parts for the area. If the perpendiculars or breadths be not at equal distances, compute all the parts separately, as so many trapezoids, and add them all together for the whole area.

EXAMPLE.

What is the area of an irregular figure, the breadths at equal distances being 8.2, 7.4, 9.2, 10.2, 8.6, and the whole length 39?

$$\begin{array}{r l} & 8.2 \\ & \underline{8.6} \\ 2) & 16.8 \text{ sum of the extremes.} \\ & 8.4 \text{ mean of the extremes.} \end{array}$$

$8.4+7.4+9.2+10.2=35.2$ sum of the breadths, now $35.2\times39=1372.8$ which divided by the number of parts $=\frac{1372.8}{4}=$ 343.2 area.

PROBLEM XIV.

To find the area of an Ellipsis or Oval.—See Fig. 12.

RULE. Multiply the longest diameter by the shortest; then multiply the product by the decimal .7854, for the area.

EXAMPLE.

What is the area of an Oval, its diameters being 7 and 5?

$$7 \times 5 \times .7854 = 27.4890 \text{ area.}$$

Note. The circumference of an Ellipsis is $\sqrt{\overline{4 D^2 + d^2}} \times 1.11$ nearly.

PROBLEM XV.

To find the area of an Elliptic Segment.

RULE 1. Find the area of a corresponding circular segment, having the same height, and the same

vertical axis or diameter; then say, as the said vertical axis is to the other axis, parallel to the segment's base; so is the area of the circular segment before found, to the area of the elliptic segment sought.

Rule 2. Divide the height of the segment by the vertical axis of the ellipse, and find in the Table of circular segments, Prob. 12, the circular segment having the above quotient for its versed sine; then multiply altogether, this segment and the two axes of the ellipse.

EXAMPLE.

What is the area of an elliptic segment, its height being 2, vertical axis 20, and parallel axis 5?

Rule 1. $20)\underline{2}$
$.1$ gives $.04088 \times 20^2 = 16.352$ area of circular segment: $20 : 5 :: 16.352 : 4.0880$ area of elliptic segment.

Rule 2. $20)\underline{2}$
$.1$ gives $.04088 \times \overline{5 \times 20} = 4.0880$ area.

PROBLEM XVI.

To find the area of a Parabola, or its Segment.
See Fig. 13.

Rule. Multiply the base by the perpendicular height; then take two-thirds of the product for the area.

EXAMPLE.

What is the area of a Parabola, its base being 6, and height 9?

$$6\times9=\frac{54\times2}{3}=36 \text{ area of segment.}$$

SOLIDS.

By Mensuration of Solids, are determined the spaces included by contiguous surfaces; and the sum of the measures of these including surfaces, is the whole surface or superficies of the body.

The measure of a body is called its solidity, capacity, or content.

Solids are measured by cubes, whose sides are inches, feet, or yards, &c; and hence the solidity of a body is said to be so many cubic inches, feet, yards, &c. as will fill its capacity or space, or another of an equal magnitude.

The least solid measure is the cubic inch; other cubes being taken from it, according to the proportion in the following Table, which is formed by cubing the linear proportions.

Table of Cubes or Solids.

1728	Cubic Inches	make	1	Cubic Foot.
27	Cubic Feet	——	1	Cubic Yard.
$166\frac{3}{8}$	Cubic Yards	——	1	Cubic Pole.
64000	Cubic Poles	——	1	Cubic Furlong.
512	Cubic Furlongs	——	1	Cubic Mile.

PROBLEM I.

To find the superficies of a Prism or Cylinder.— See Fig. 14 and 15.

Rule. Multiply the perimeter of one end of the Prism, by the length of the solid, and the product will be the surface of all its sides. To which add also the area of the two ends of the Prism, when required.

Or, compute the areas of all the sides and ends separately, and add them all together.

EXAMPLE.

What is the superficies of an equilateral triangular Prism, its length being 9, and side 3?

$3 \times 3=9$ perimeter, then $9 \times 9=81$ superficies, or $3 \times 9=27$ area of one side, and 27×3 81 superficies or areas of the 3 sides.

PROBLEM II.

To find the surface of a Pyramid or Cone.—See Fig. 16 and 17.

RULE. Multiply the perimeter of the base by the slant height, or length of the side, and half the product will be the surface of the sides, or the sum of the areas of all the triangles which form it. To which add the area of the end or base, if required.

EXAMPLE.

What is the surface of a Pyramid, its slant height being 20, and the perimeter of its base 15?

$$\frac{15 \times 20}{2} = 150 \text{ surface of pyramid.}$$

PROBLEM III.

To find the surface of the Frustum of a Pyramid or Cone, being the lower part, when the top is cut off by a plane parallel to the base.

RULE. Add together the perimeters of the two ends, and multiply their sum by the slant height, taking half the product for the answer—As is evident, because the sides of the solid are trapezoids, having the opposite sides parallel.

EXAMPLE.

What is the surface of the frustum of a Cone, the slant height being 12, the diameters 8 and 6?

$$3.1416 \times 8 = 25.12$$
$$3.1416 \times 6 = \underline{18.85}$$
$$43.97 \text{ perimeters of both ends,}$$

$43.97 \times 12 = 527.64$, which halved, is 263.82, the surface of frustum.

PROBLEM IV.

To find the solid content of any Prism or Cylinder.

Find the area of the base, or end, whatever the figure of it may be, and multiply it by the length of the Prism or Cylinder, for the solid content.

EXAMPLE.

What is the solid content of a Cylinder, its diameter being 3, and length 7?

$.7854 \times 9 = 7.0686$ area, $7.0686 \times 7 = 49.4802$ solidity.

PROBLEM V.

To find the content of any Pyramid or Cone.— See Fig. 16 and 17.

RULE. Find the area of the base, and multiply that area by the perpendicular height; then take one third of the product for the content.

EXAMPLE.

What is the content of a Cone, the area of its base being 9, and vertical height 17?

$$17 \times 9 = 153, \frac{153}{3} = 51 \text{ solid content.}$$

PROBLEM VI.

To find the solidity of the Frustum of a Cone or Pyramid.

Rule. Add into one sum the areas of the two ends, and the mean proportional between them; and take one-third of that sum for a mean area; which being multiplied by the perpendicular height or length of the frustum, will give its content.

EXAMPLE.

What is the solidity of the frustum of a Cone, the vertical height 19, the areas of its ends being 12 and 9?

$$\begin{array}{rl} & 12 \\ & 9 \\ & 10.5 \text{ mean proportional.} \\ 3) & \overline{31.5} \\ & \overline{10.5} \times 19 = 199.5 \text{ solid content.} \end{array}$$

PROBLEM VII.

To find the surface of a Sphere or any Segment. See Fig. 18.

Rule 1. Multiply the circumference of the Sphere by its diameter, and the product will be the whole surface of it.

RULE 2. Square the diameter, and multiply that square by 3.1416, for the surface.

RULE 3. Square the circumference; then either multiply that square by the decimal .3183, or divide it by 3.1416, for the surface.

Note. For the surface of a Segment or Frustum, multiply the whole circumference of the Sphere by the height of the part required.

EXAMPLE.

What is the superficial content of a Sphere, its diameter being 7?

Rule 1. $\left\{\begin{array}{l}\text{Circum. } 22\\ \text{Diam}^{\text{r.}}\ 7\end{array}\right.$ $22 \times 7 = 154$ superf. cont.

Rule 2. $7^2 \times 3.1416 = 153.9384$ do.

Rule 3. $\left\{\begin{array}{l}22^2 \times .3183 = 154.0572 \quad \text{do.}\\ \text{or} \quad \dfrac{22^2}{3.1416} = 154.06 \quad \text{do.}\end{array}\right.$

PROBLEM VIII.

To find the solidity of a Sphere or Globe.

RULE 1. Multiply the surface by the diameter, and take 1-6th of the product for the content; or, which is the same thing, multiply the square of the diameter by the circumference, and take 1-6th of the product.

Rule 2. Take the cube of the diameter, and multiply it by the decimal .5236, for the content.

Rule 3. Cube the circumference, and multiply it by .01688, for the content.

EXAMPLE.

What is the solid content of a Globe 7 inches diameter?

This is the same diameter of the Sphere as in last Example; therefore the surface will be 154 inches.

Rule 1. $\frac{154\times7}{6}=179\frac{2}{3}$ or $\frac{7^2\times22}{6}=179\frac{2}{3}$ cub. cont.

Rule 2. $7^3=343\times.5236=179.5948$ cub. content.

Rule 3. $22^3=10648\times.01688=179.73824$ do.

PROBLEM IX.

To find the solid content of a Spherical Segment.
See Fig. 18.

Rule 1. From 3 times the diameter of the Sphere, take double the height of the segment; then multiply the remainder by the square of the height, and the product by the decimal .5236, for the content.

Rule 2. To 3 times the square of the radius of the Segment's base, add the square of its height;

then multiply the sum by the height, and the product by .5236, for the content.

EXAMPLE.

What is the solid content of a Spherical Segment 2 feet high, taken from a Sphere 8 feet diameter ?

Rule 1. $8\times3=24-2\times2=20\times2^2=80\times.5236=41.888$, radius of sphere is 4, now $4^2=16$, and from the radius take the height of the segment, $4-2=2$, which $2^2=4$, therefore $16-4=12=$square of the radius of Segment's base.

Rule 2. $\overline{12\times3+2^2}\times2\times.5236=41.888$ content.

In arranging the first Edition, it was thought superfluous to give Examples for the elucidation of the preceding very simple Rules; but after publication, it was found that the want of Examples operated against the general usefulness of the book ; for this reason they are now annexed.

SPECIFIC GRAVITY.

THE specific gravity of a body is the proportional weight between that body and another of a known density; and water is admirably adapted to be the standard, as a solid foot of it weighs 1000 ounces Avoirdupois.

TO FIND THE SPECIFIC GRAVITY OF A BODY.

PROBLEM I.

When the body is heavier than water.

RULE. Weigh it both in and out of water, and take the difference, which will be the weight lost in water; then say,

As the weight lost in water,
Is to the whole or absolute weight;
So is the specific gravity of water,
To the specific gravity of the body.

EXAMPLE.

What is the specific gravity of a stone which weighs 10 libs, but in the water only $6\frac{3}{4}$ libs, water being 1000?

$10-6\frac{3}{4}=3\frac{1}{4}$ weight lost in water.
$3\frac{1}{4}$: 10 : : 1000 : 3077 specific gravity.

PROBLEM II.

When the body is lighter than water.

RULE. Annex to it a piece of another body heavier than water, so that the mass compounded of the two may sink together. Weigh the denser body and the compound mass separately, both in and out of water, then find how much each loses in water, by subtracting its weight in water from its weight in air, and subtract the less of these remainders from the greater; then say,

As the last remainder,
Is to the weight of the light body in air;
So is the specific gravity of water,
To the specific gravity of the body.

EXAMPLE.

What is the specific gravity of a piece of elm which weighs in air 15 libs; attached to it is a piece

of copper, weighing 18 libs. in air, and 16 libs. in water, and this compound weighs in water 6 libs?

$$\begin{array}{l} 33 = 18 + 15 \\ \underline{6} \quad \underline{16} \\ 27 - 2 = 25 \textit{ last remainder.} \end{array}$$

As 25 : 15 : : 1000 : 600, specific gravity of the elm.

PROBLEM III.

For a Fluid of any sort.

RULE. Take a piece of a body of known specific gravity, weigh it both in and out of the fluid, finding the loss of weight by taking the difference of the two; then say,

As the whole or absolute weight,
Is to the loss of weight;
So is the specific gravity of the solid,
To the specific gravity of the fluid

EXAMPLE.

What is the specific gravity of fluid in which a piece of cast iron weighs 34.61 oz. and 40 oz. out of it?

40 : 5.39 : : 7425 : 1000, specific gravity of the fluid.

Another method.

RULE. Weigh any convenient body, (a bubble of glass is best for the purpose,) in air, in water, and in the fluid whose specific gravity is required; then say,

As the loss of weight in water,
Is to the loss of weight in the other fluid;
So is the specific gravity of water,
To the specific gravity of the fluid.

EXAMPLE.

A body weighs 1000 grs. in air, 750 grs. in water, and 770 in a liquid whose specific gravity is required.

250 : 230 : : 1000 : 920, specific gravity of the liquid.

PROBLEM IV.

To find the quantities of two ingredients in a given compound.

Rule. Take the three differences of every pair of the three specific gravities, namely, the specific gravities of the compound and each ingredient; and multiply each specific gravity by the difference of the other two; then say,

As the greatest product,
Is to the whole weight of the compound,
So is each of the other two products,
To the weights of the two ingredients.

EXAMPLE.

A composition of 112 libs. being made of tin and copper, whose specific gravity is found to be 8784; required the quantity of each ingredient, the specific gravity of tin being 7320, and copper 9000.

8784,	. . .	Composition,
9000,	. . .	Copper,
7320,	. . .	Tin.

$$9000-7320=1680\times8784=14757120$$
$$8784-7320=1464\times9000=13176000$$
$$9000-8784=\ 214\times7320=\ 1566480$$

$$\left.\begin{array}{r}\text{As } 14757120:112::13176000:100=\text{Copper}\\ 112-100=12=\text{Tin}\end{array}\right\}\text{Weight of Ingredients}$$

A Table of Specific Gravities of Bodies.

Platina (pure)	23000	Tin	7320
Fine Gold	19400	Clear Crystal Glass . .	3150
Standard Gold	17724	Granite	3000
Quicksilver (pure) . .	14000	Marble and Hard Stone	2700
Quicksilver (common) .	13600	Common Green Glass .	2600
Lead	11325	Flint	2570
Fine Silver	11091	Common Stone	2520
Standard Silver	10535	Clay	2160
Copper	9000	Brick	2000
Copper Halfpence . . .	8915	Common Earth	1984
Gun Metal	8784	Nitre	1900
Cast Brass	8000	Ivory	1825
Steel	7850	Brimstone	1810
Iron	7645	Solid Gunpowder . .	1745
Cast Iron	7425	Sand	1520
Coal	1250	Ash	800
Boxwood	1030	Maple	755
Sea Water	1030	Elm	600
Common Water . . .	1000	Fir	550
Oak	925	Charcoal	400
Gunpowder close shaken	937	Cork	240
Do. in a loose heap	836	Air at a mean state . .	$1\frac{2}{9}$

Note. The several sorts of wood are supposed to be dry. Also, as a cubic foot of water weighs just 1000 ounces avoirdupois, the numbers in this Table

express, not only the specific gravities of the several bodies, but also the weight of a cubic foot of each in avoirdupois ounces; and therefore, by proportion, the weight of any other quantity, or the quantity of any other weight, may be known. Also, 100 cubic inches of common air weigh nearly $31\frac{1}{4}$ grains troy, or $1\frac{1}{7}$ drams avoirdupois. HUTTON.

It is more usual to take the specific gravity of water as unity. The preceding table will give the specific gravity of the substances mentioned in it, by placing the decimal point before the three last figures of each of the numbers.

TABLES OF THE WEIGHT OF MALLEABLE AND CAST IRON PLATES, BARS, &c.

TABLE *of the Weight of a Square Foot of Cast and Malleable Iron, Copper and Lead, from* 1-16*th to* 1 *Inch thick.*

Thick.	Cast Iron		Mall. Iron.		Copper.		Lead.	
	Libs.	*Oz.*	*Libs.*	*Oz.*	*Libs.*	*Oz.*	*Libs.*	*Oz.*
1 Sixteenth	2	6.6	2	7.8	2	15	3	11
2 —	4	13.3	4	15.6	5	14	7	6
3 —	7	4.	7	7.4	8	13	11	1
4 —	9	10.6	9	15.2	11	12	14	12
5 —	12	1.3	12	7.1	14	11	18	7
6 —	14	8.	14	14.9	17	10	22	2
7 —	16	14.7	17	6.7	20	9	25	13
8 —	19	5.3	19	14.5	23	8	29	8
9 —	21	12.	22	6.3	26	7	33	3
10 —	24	2.7	24	14.2	29	6	36	14
11 —	26	9.3	27	6.	32	5	40	9
12 —	29	–	29	13.8	35	4	44	4
13 —	31	6.7	32	5.6	38	3	47	15
14 —	33	13.4	34	13.4	41	2	51	10
15 —	36	4.	37	5.3	44	1	55	5
1 Inch	38	10.7	39	13.1	47	–	59	–

TABLE *of the Weight of a Lineal Foot of Malleable and Cast Iron Bars, from 6-16ths to 3 Inches square.*

Sixteenths on the side.	Area in Square Sixteenths.	MALL. IRON. *Ounces Weight.*	CAST IRON. *Ounces Weight*	ROUND RODS. The 16ths on the side is the diameter of Rod. *Ounces Weight.*
6	36	7.4736	. . .	5.83
7	49	10.1724	. . .	7.99
8	64	13.2864	12.8960	10.43
9	81	16.8156	. . .	13.20
10	100	20.7600	. . .	16.30
11	121	25.1196	. . .	19.72
12	144	29.8944	29.0160	23.47
13	169	35.0844	. . .	27.53
14	196	40.6896	. . .	31.94
15	225	46.7100	. . .	36.44
1 Inch	256	53.1456	51.5840	41.50
1	289	59.9964	. . .	46.80
2	324	67.2624	. . .	52.47
3	361	74.9436	. . .	58.46
4	400	83.0400	80.6000	64.81
5	441	91.5516	. . .	71.41
6	484	100.4784	. . .	78.37
7	529	109.8204	. . .	85.66
8	576	119.5774	116.0640	93.27
9	625	129.7500	. . .	101.21
10	676	140.3376	. . .	109.46
11	729	151.3404	. . .	118.05
12	784	162.7584	157.9760	126.95
13	841	174.5916	. . .	136.19
14	900	186.8400	. . .	145.74
15	961	199.5036	. . .	155.62
2 Inches	1024	212.5824	206.3360	165.82
1	1089	226.0764	. . .	176.34
2	1156	239.9856	. . .	187.19
3	1225	254.3100	. . .	198.36
4	1296	269.0496	261.1440	209.86
5	1369	284.2044	. . .	221.68
6	1444	299.7744	. . .	233.83
7	1521	315.7596	. . .	246.30
8	1600	332.1600	322.4000	259.09
9	1681	348.9756	. . .	272.20
10	1764	366.2064	. . .	285.64
11	1849	383.8524	. . .	299.41
12	1936	401.9136	390.1040	313.49
13	2025	420.3900	. . .	327.91
14	2116	439.2816	. . .	342.64
15	2209	458.5884	. . .	357.70
3 Inches	2304	478.3104	464.2560	373.09

The foregoing Tables have been calculated from Hutton's Specific Gravities; those of Cast and Malleable Iron and Lead agree very nearly with those given by other authors; but the specific gravity of Copper, though heavier than that given by Hatchett, which is 8.800; still, from Copper being frequently alloyed with Lead, it is supposed that Hutton's, which is 9000, will be nearest the weight of Copper commonly used.

As water varies in bulk at different temperatures, it becomes necessary in some calculations to take into view this circumstance. For this purpose, the approximate specific gravity obtained by the foregoing rules must be multiplied by the specific gravity of water of the temperature at which the experiment is performed, the specific gravity of water at the standard temperature being taken as the unit. The best temperature for a standard is one pointed out by a physical fact in the constitution of water independent of thermometrical indications. Water contracts only until it is cooled to the temperature of 40°, it then remains of the same bulk until cooled to 38°, and afterwards begins to expand, which it continues to do until it freezes. To facilitate reductions to this temperature a table is subjoined.

TABLE *of the Specific Gravity of Water at various temperatures by Fahrenheit's Thermometer.*

TEMPERATURE.	SPEC. GRAVITY.	EXP. FOR 1°.
30°	999.82	0.04
32°	999.89	0.03
34°	999.95	0.02
38° to 40°	1000.00	0.00
45°	999.91	0.02
50°	999.77	0.04
60°	999.08	0.08
70°	998.02	0.12
80°	996.66	0.16
90°	994.91	0.20
100°	992.89	0.24
120°	988.04	0.29
140°	982.44	0.31
160°	976.29	0.34
180°	969.72	0.36
200°	962.62	0.37
212°	958.60	0.38

FALLING BODIES.

The motion described by Bodies freely descending by their own gravity, is, viz.—The Velocities are as the Times, and the Spaces as the Squares of the Times.—Therefore if the Times be as the

numbers	1	2	3	4	&c.
The Velocities will be also as .	1	2	3	4	&c.
The Spaces as their Squares . .	1	4	9	16	&c.
and the Spaces for each time, as	1	3	5	7	&c.

namely, as the series of the odd numbers, which are the differences of the squares, denoting the whole spaces:—So that if the first series of numbers

be seconds of time: *i. e.* . . .	1″	2″	3″	&c.
Velocities in feet will be . .	$32\frac{1}{6}$	$64\frac{1}{3}$	$96\frac{1}{2}$	&c.
Spaces in the whole times will be	$16\frac{1}{12}$	$64\frac{1}{3}$	$144\frac{3}{4}$	&c.
Spaces for each second will be	$16\frac{1}{12}$	$48\frac{1}{4}$	$80\frac{5}{12}$	&c.

HUTTON.

The following Table *shows the Spaces fallen through, and the Velocities acquired, at the end of each of* 30 *Seconds.*

Time in Seconds	SPACE.				VELOCITY.		
	Each Time.	As the Squares of the Time.	Fallen through in Feet & Inches.		As the Times.	Acquired in Feet & Inches.	
1	1	1	16	1	1	32	2
2	3	4	64	4	2	64	4
3	5	9	144	9	3	96	6
4	7	16	257	4	4	128	8
5	9	25	402	1	5	160	10
6	11	36	579	0	6	193	0
7	13	49	788	1	7	225	2
8	15	64	1029	4	8	257	4
9	17	81	1302	9	9	289	6
10	19	100	1608	4	10	321	8
11	21	121	1946	1	11	353	10
12	23	144	2316	0	12	386	0
13	25	169	2718	1	13	418	2
14	27	196	3152	4	14	450	4
15	29	225	3618	9	15	482	6
16	31	256	4117	4	16	514	8
17	33	289	4648	1	17	546	10
18	35	324	5211	0	18	579	0
19	37	361	5806	1	19	611	2
20	39	400	6433	4	20	643	4
21	41	441	7092	9	21	675	6
22	43	484	7784	4	22	707	8
23	45	529	8508	1	23	739	10
24	47	576	9264	0	24	772	0
25	49	625	10052	1	25	804	2
26	51	676	10872	4	26	836	4
27	53	729	11724	9	27	868	6
28	55	784	12609	4	28	900	8
29	57	841	13526	1	29	932	10
30	59	900	14475	0	30	965	0

EXAMPLE I.

To find the space descended by a body in 7″ and the velocity acquired.

$$\begin{array}{l} 16\ 1 \times 49 = \overset{ft.}{788}\ \overset{in.}{1} \text{ of space.} \\ 32\ 2 \times 7'' = 225\ 2 \text{ of velocity.} \end{array}$$

Look into the Table at 7″ and you have the answers.

EXAMPLE II.

To find the time of generating a velocity of 100 feet per second, and the whole space descended.

$$\frac{100 \quad \times 12}{32\ \ 2 \times 12} = 3''\tfrac{21}{193} \text{ Time.}$$

$$\frac{3''\tfrac{21}{193} \times 100}{2} = 155\tfrac{85}{193} \text{ Space descended.}$$

EXAMPLE III.

To find the time of descending 400 feet, and the velocity at the end of that time.

$$\sqrt{\frac{400 \quad \times 12}{16\ 1 \times 12}} = 4''\ 987 \text{ Time.}$$

$$\frac{400 \quad \times 2}{4''\ 987} = 169.662 \text{ Velocity.}$$

Or these answers can be found from the Table by Proportion.

PENDULUM.

The vibrations of Pendulums are as the square roots of their lengths; and as it has been found by many accurate experiments, that the pendulum vibrating seconds in the latitude of London, is $39\frac{1}{8}$ inches long nearly, the length of any other pendulum may be found by the following Rule, viz. As the number of vibrations given, is to 60, so is the square root of the length of the pendulum that vibrates seconds, to the square root of the length of the pendulum that will oscillate the given number of vibrations:—or, As the square root of the length of the pendulum given, is to the square root of the length of the pendulum that vibrates seconds, so is 60 to the number of vibrations of the given pendulum.

Since the pendulum that vibrates seconds, or 60, is $39\frac{1}{8}$ inches long, the calculation is rendered simple; for $\sqrt{39\frac{1}{8}} \times 60 = 375$, a constant number, therefore 375, divided by the square root of the pendulum's length, gives the vibrations per minute, and divided by the vibrations per minute, gives the square root of the length of the pendulum.

EXAMPLE. I.

How many vibrations will a pendulum of 49 inches long make in a minute?

$$\frac{375}{\sqrt{49}} = 53\tfrac{4}{7} \text{ vibrations in a minute.}$$

EXAMPLE II.

What length of a pendulum will it require, to make 90 vibrations in a minute?

$$\frac{375}{90}=4.16 \quad \overline{4.16}^2 = 17.3056 \text{ inches long.}$$

EXAMPLE III.

What is the length of a pendulum, whose vibrations will be the same number as the inches in its length?

$$\sqrt[3]{375^2} = 52 \text{ inches long, and } 52 \text{ vibrations.*}$$

It is proposed to determine the length of a pendulum vibrating seconds, in the latitude of London, where a heavy body falls through $16\frac{1}{12}$ feet in the first second of time?

---

* For $\sqrt{x} : \sqrt{39\frac{1}{8}} :: 60 : x$ or $\sqrt{x} \times x = 375$

$$\sqrt{x} = \frac{375}{x} = x = \frac{\overline{375^2}}{x^2}$$

$$\text{or } x \times x^2 = \overline{375^2} = x^3 = \overline{375^2}$$

$$x = \sqrt[3]{\overline{375^2}} = 52.002$$

3.1416 circumference, the diameter being 1. }
$16\frac{1}{12}$ feet = 193 inches fall in the 1″ of time. } *

$$\begin{array}{l} 193 \times 2 = 386.00000000 \\ 3.1416^2 = 9.86965056 \end{array} = 39.109 \text{ inches,}$$

or 39.11 inches.

By experiment this length is found to be $39\frac{1}{8}$ inches.

What is the length of a pendulum vibrating in 2 seconds, and another in half a second?

$\sqrt{39\frac{1}{8}} = 6.25 \times 60 = 375.$

$\frac{375}{30} = 12.5$ squared $= 156.25$ inches the length of [a 2 seconds' Pendulum.

$\frac{375}{120} = 3.125$ squared $= 9.765625$ inches the length of a $\frac{1}{2}$ second's Pendulum.

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* When a pendulum vibrates in a Cycloid; the time of one vibration, is to the time a body falls through half the length of the pendulum, as the circumference of a circle, is to its diameter. The times in which bodies descend through similar parts of similar curves are as the square roots of their lengths. Therefore $\sqrt{\frac{1}{2}l} : \sqrt{g} :: 1'' : \sqrt{\frac{2g}{l}}$ l being = length and g equal to the fall in 1″ now as $1 : 3.1416 :: \sqrt{\frac{2g}{l}} : 3.1416 \times \sqrt{\frac{2g}{l}}$ let $3.1416 = p$ then $p^2 = \frac{2g}{l}$. $\frac{1}{2}p^2 \times l = g$. $\frac{g}{\frac{1}{2}p^2} = l$.

MECHANICAL POWERS, &c.

The Science of Mechanics is simply the application of Weight and Power, or Force and Resistance. The weight is the resistance to be overcome; the power is the force requisite to overcome that resistance. When the force is equal to the resistance, they are in a state of equilibrium, and no motion can take place; but when the force becomes greater than the resistance, they are not in a state of equilibrium, and motion takes place; consequently, the greater the force is to the resistance, the greater is the motion or velocity.

The Science of Equilibrium is called STATICS; the Science of Motion is called DYNAMICS.

Mechanical Powers are the most simple of mechanical applications to increase force, and overcome resistance. They are usually accounted six in number, viz. The LEVER,—THE WHEEL and AXLE,—The PULLEY,—The INCLINED PLANE,—The WEDGE,—and the SCREW.

LEVER.

To make the principle easily understood, we must suppose the Lever an inflexible rod without weight; when this is done, the rule to find the equilibrium between the power and the weight, is,—Multiply the weight by its distance from the fulcrum, prop, or centre of motion, and the power by its distance from the same point: if the products are equal, the weight and power are in equilibrio; if not, they are to each other as their products.

EXAMPLE I.

A weight of 100 libs on one end of a lever, is 6 inches from the prop, and the weight of 20 libs at the other end, is 25 inches from the prop—What additional weight must be added to the 20 libs, to make it balance the 100 libs?

$\frac{100 \times 6}{25} = 24 - 20 = 4$ libs weight to be added.

EXAMPLE II.

A Block of 960 libs is to be lifted by a lever 30 feet long, and the power to be applied is 60 libs—on what part of the lever must the fulcrum be placed?

$\frac{960}{60} = 16$, that is, the weight is to the power as 16 is to 1; therefore the whole length $\frac{30}{16+1} = 1\frac{13}{17}$, the distance from the block, and $30 - 1\frac{13}{17} = 28\frac{4}{17}$, the distance from the power.

EXAMPLE III.

A Beam 32 feet long, and supported at both ends, bears a weight of 6 tons, 12 feet from one end,—What proportion of weight does each of the supports bear?

$\frac{12 \times 6}{32} = 2\frac{1}{4}$ tons, support at end farthest from the weight.

$\frac{20 \times 6}{32} = 3\frac{3}{4}$ tons, support at end nearest the weight.

EXAMPLE. IV.

A Beam supported at both ends, and 16 feet long, carries a weight of 6 tons, 3 feet from one end, and another weight of 4 tons, 2 feet from the other end: What proportion of weight does each of the supports bear?

$\frac{3 \times 6}{16} + \frac{14 \times 4}{16} = \frac{74}{16} = 4\frac{10}{16}$ tons, end at the 4 tons.

$\frac{2 \times 4}{16} + \frac{13 \times 6}{16} = \frac{86}{16} = 5\frac{6}{16}$ tons, end at the 6 tons.

When the weight of the lever is taken to account, *see Centre of Gravity.*

WHEEL AND AXLE.

See Fig. 1. *Plate* 2.

The power gained by the Wheel and Axle, Wheel and Pinion, or Crane, is the effect of a double lever; for suppose the end of the lever *a b* is the radius of the rope barrel, and the radius of the wheel *b c;* again, the radius of the pinion working into the wheel, is *d e,* and the length of the handle or winch is *e f.*

If the distance between *a b* is only one-third of the distance between *b c,* it is evident, that the point at *c* will go through three times the space to that of the point at *a,* when the lever revolves round its fulcrum *b:* the points *d* & *f,* in the other lever, are in the same proportion. The short end *d* acts upon the long end *c:* and if the end *f* goes through 9 inches, the end *d* will go through 3 inches, also the end *c.* If the end *c* goes through 3 inches, the end *a* will go through only 1 inch; therefore the power is to the weight as 9 is to 1; that is, If 9 libs be hung at the end of the arm *a,* and 1 lib hung at the end of the arm *f,* they will balance each other. From this it is evident, that if you gain power you lose speed; and by gaining speed, you lose power: hence the Rule is deduced—Multiply the power applied by its velocity, and the weight to be raised by its velocity.

Note. A man's power producing the greatest effect, is 31 libs at a velocity of 2 feet per second, or 120 feet per minute.

The Rule to find the power of Cranes is, viz.

Divide the product of the driven by the product of the drivers, and the quotient is the relative velocity, as 1: v, which multiplied by the length of winch, and by the power applied (in libs.) and divided by the radius of the barrel, the quotient will be the weight raised.

$$\begin{array}{l} \text{Product of drivers} = a \\ \text{Do.} \quad \text{of driven} = b \end{array} \quad \text{and} \quad \frac{b}{a} = v$$

Length of winch . .	$= w$	$\frac{vwp}{n} = R$
Radius of barrel . .	$= n$	$Rn = vwp$
Weight or Power applied	$= p$	$\frac{Rn}{vw} = p$
Weight raised . . .	$= R$	

EXAMPLE I.

A weight of 94 tons is to be raised 360 feet in 15 minutes, by a power, the velocity of which is 220 feet per minute:—What is the power required?

$\frac{360}{15} = 24$ feet per minute, velocity of weight.

$24 \times 94 = \frac{2256}{220} = 10.25\dot{4}\dot{5}$ tons power required.

EXAMPLE II.

A Stone weighing 986 libs, is required to be lifted: What power must be applied, when the power is to the weight as 9 is to 2?

$$\frac{986 \times 2}{9} = \frac{1972}{9} = 219\tfrac{1}{9} \text{ tons power.}$$

EXAMPLE III.

A Power of 18 libs is applied to the winch of a crane, the length of which is 8 inches; the pinion makes 12 revolutions for 1 of the wheel, and the barrel is 6 inches diameter.

$\dfrac{8 \times 2 \times 22}{7} = 50.28$ circumference of the winch's circle.

$50.28 \times 12 = 603.36$ inches velocity of power on winch to 1 revolution of the barrel.

$$\frac{603.36 \times 18 = 10860.48}{\frac{6 \times 22}{7} = 18.857\,.\,.\,19.} = 571.604 \text{ libs weight,}$$

that can be raised by a power of 18 libs on this crane.

PULLEY.

There are two kinds of Pulleys, the *fixed* and the *moveable*. From the fixed Pulley no power is derived; it is as a common beam used in weighing goods, having the two ends of equal weight, and at

the same distance from the centre of motion: the only advantage gained by the fixed pulley, is in changing the direction of the power.

From the moveable pulley power is gained; it operates as a lever of the second order; for if one end of a string be fixed to an immoveable stud, and the other end to a moveable power, the string doubled and the ends parallel, the pulley that hangs between is a lever; the fixed end of the string being the fulcrum, and the other the moveable end of the lever: hence the power is double the distance from the fulcrum, than is the weight hung at the pulley; and therefore the power is to the weight as 2 is to 1. This is all the advantage gained by one moveable pulley, for two, twice the advantage; for three, thrice the advantage; and so on for every additional moveable pulley.

From this the following Rule is derived:—Divide the weight to be raised by twice the number of moveable pulleys, and the quotient is the power required to raise the weight.

EXAMPLE I.

What power is requisite to lift 100 libs, when two blocks of three pulleys, or sheives each, are applied, the one block moveable and the other fixed?

$\frac{100}{6} = 16\frac{4}{6}$ libs, the power required,

3 sheives $\times$ 2 $=$ 6.

EXAMPLE II.

What weight will a power of 80 libs lift, when applied to a 4 and 5 sheived block and tackle, the 4 sheived block being moveable?*

$$80 \times 8 = 640 \text{ libs weight raised.}$$

INCLINED PLAIN.

When a body is drawn up a vertical plane, the whole weight of the body is sustained by the power that draws or lifts it up: hence the power is equal to the weight.

When a body is drawn along an horizontal (truly level) plane, it takes no power to draw it (save the friction occasioned by the rubbing along the plane.)

From these two hypotheses, if a body is drawn up an inclined plane, the power required to raise it is as the inclination of the plane; and hence when the power acts parallel to the plane, the length of the

* *See Fig.* 2, *Plate* 2.—If the strings be not parallel, but in the directions D A, D B, then the power A requisite to lift the weight C is as D E is to D C, and the strain upon the fixed point B is as C E is to C D.—HUTTON.

From this simple definition, it is easy to find the proportion between the power and the weight, when the strings are at any angle.

plane is to the weight, as the height of the plane is to the power; for the greater the angle, the greater the height.

EXAMPLE I.

What power is requisite to move a weight of 100 libs up an inclined plane, 6 feet long and 4 feet high?

If $6 : 4 :: 100 : 66\frac{2}{3}$ libs power.

EXAMPLE II.

A power of 68 libs, at the rate of 200 feet per minute, is applied to pull a weight up an inclined plane, at the rate of 50 feet per minute—When the plane is 37 feet long and 12 feet high, how much will be the weight drawn?*

As $12 : 37 :: 68 \times 200 : 50 \times 838^4$

$$\frac{68 \times 200 \times 37}{12 \times 50} = \frac{503200}{600} = 838\tfrac{4}{6} \text{ libs weight.}$$

WEDGE.

The Wedge is a double inclined plane, and therefore subject to the same Rules; or the following Rule, which is particularly for the Wedge; but

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* *See Fig.* 3, *Plate* 2. When the direction of the power is at any angle than parallel to the plane.

Draw B C at right angles to the direction of the power P, then the Weight is to the Power as A D is to D C—that is, W : P : : A D : D C.

drawn from its near connexion to the inclined plane, is,—If the power acts perpendicularly upon the head of the wedge, the power is to the pressure which it exerts perpendicularly on each side of the wedge, as the head of the wedge is to its side: hence, it is evident, that the sharper or thinner the wedge is, the greater will be the power.

But the power of the Wedge being not directly according to its length and thickness, but to the length and width of the split, or rift, in the wood to be cleft, the rule therefore is of little use in practice; besides, the wedge is very seldom used as a power; for these reasons, the nature of its properties and effects need not be here discussed.

SCREW.

The Screw is a cord wound in a spiral direction round the periphery of a cylinder, and is therefore an inclined plane, the length being the circumference of the cylinder, and the height, the distance between two consecutive cords, or threads of the Screw, hence, the Rule is derived:—As the circumference of the Screw is to the Pitch, or distance between the threads; so is the Weight to the Power.

When the Screw turns, the cord or thread runs in a continued ascending line round the centre of the cylinder, and the greater the radius of the cylin-

der, the greater will be the length of the plane to its height, consequently, the greater the power.—A lever fixed to the end of the screw will act as one of the second order, and the power gained will be as its length, to the radius of the cylinder; or the circumference of the circle described by it, to the circumference of the cylinder; hence, an addition to the rule is produced, which is,—If a lever is used, the circumference of the lever is taken for, or instead of, the circumference of the screw.

EXAMPLE I.

What is the power requisite to raise a weight of 8000 libs by a screw of 12 inches circumference and 1 inch pitch?

As 12 : 1 : : 8000 : $666\frac{8}{12}$ libs = power at the circumference of the screw.

EXAMPLE II.

How much would be the power if a lever of 30 inches was applied to the screw?

Circumference of 30 inches=$188\frac{4}{7}$

As $188\frac{4}{7}$: 1 : : 8000 : $42\frac{560}{1320}$ libs = power with a lever of 30 inches long.

STRENGTH OF MATERIALS.

After considering the Mechanical Powers, which are the analytic parts of all Machines, the next step is to consider the Strength of the Materials of which Machinery is composed—this knowledge being of the greatest importance to the Mechanic, by enabling him to adjust, with respect to magnitude, the various parts of the machine, that the strength of each part may be proportional to the stress it has to sustain.

COHESIVE STENGTH OF BEAMS, BARS, &c.

The cohesive strength of a body, is that force by which its fibres or particles resist separation, therefore the more particles that are in a body, the greater will be the power requisite to tear them

asunder; or, according to the rule, that the strength of bodies are as the area of their cross sections.

The knowledge in this property of bodies is very limited, there being very few experiments made on which to build a data, and these few do not agree.

The following are the results of experiments made by Mr. Emerson, which state the load that may be safely borne by a square inch rod of each.

	Pounds Avoirdupois.
Iron rod, an inch square, will bear	76,400
Brass	35,600
Hempen Rope	19,600
Ivory	15,700
Oak, Box, Yew, Plum-tree . . .	7,850
Elm, Ash, Beech	6,070
Walnut, Plum	5,360
Red Fir, Holly, Elder, Plane, Crab .	5,000
Cherry, Hazel	4,760
Alder, Asp, Birch, Willow . . .	4,290
Lead	430
Free Stone	914

Mr. Barlow's opinion of this table, is, "We shall only observe here, that they all fall very short of the ultimate strength of the woods to which they refer." *See Barlow's Essay on the strength of Timber. Art.* 3.

Mr. Emerson also gives the following practical Rule, viz. "That a cylinder whose diameter is *d* inches, loaded to one fourth of its absolute strength, will carry as follows:

	Cwt.
Iron, . . .	$135 \times d^2$
Good Rope, .	$22 \times d^2$
Oak, . . .	$14 \times d^2$
Fir, ...	$9 \times d^2$

Captain S. Brown made an experiment on Welsh Pig Iron, and the result is described as follows:

"A Bar of Cast Iron, Welsh Pig, 1¼ inch square, 3 feet 6 inches long, required a strain of 11 tons, 7 cwt. (25,424 libs,) to tear it asunder, broke exactly transverse, without being reduced in any part; quite cold when broken, particles fine, dark bluish grey colour."—From this experiment, it appears that 16,265 libs. will tear asunder a square inch of Cast Iron.

Mr. G. Rennie also made some experiments on Cast Iron, and the result was, "that a Bar one inch square, cast horizontal, will support a weight of 18,656 libs—and one cast vertical, will support a weight of 19,488 libs."

There have been several experiments made on Malleable Iron, of various qualities, by different Engineers.

The mean of Mr. Telford's experimente, is 29¼ tons.
The mean of Capt. S. Brown's do. is 25 do.
and the mean between these two means, is 27 tons, nearly; which may be assumed as the medium strength of a Malleable Iron Bar 1 inch square,
See Barlow's Essay, page 235.

PROBLEM I.

To find the ultimate transverse strength of any Rectangular Beam of Timber, fixed at one end, and loaded at the other.

Rule. Multiply the number in the table of Multiplicands, by the breadth and square of the depth, both in inches, and divide that product by the length also in inches, the quotient will be the weight in libs.*

EXAMPLE I.

What weight will it require to break a beam of Fir, the breadth being 2 inches, depth 6 inches, and length 20 feet?

$$\frac{1100 \times 36 \times 2}{240} = 330 \text{ libs.}$$

EXAMPLE II.

What is the weight requisite to break a beam of Ash 7 inches square, 3 feet from the wall?

$$\frac{2026 \times 7^3}{36} = 19303\tfrac{10}{36} \text{ libs.}$$

* When the beam is loaded uniformly throughout its length, the same rule will still apply, only the result must be doubled.

EXAMPLE III.

What will be the dimensions of a Fir Beam 26 feet long, to support a weight of 400 libs?

$\frac{312 \times 400}{1100} = 113.5$ the breadth and square of the depth.

Suppose the breadth to be 2½ inches, then $\frac{113.5}{2.5} = 51.4$ the square of the depth, and $\sqrt{51.4} = 7.17$ the depth.

Suppose the depth to be 8 inches, then $8^2 = \frac{113.5}{64} = 1.77$ the breadth.

PROBLEM II.

To compute the ultimate transverse strength of any Rectangular Beam, when suppcrted at both ends, and loaded in the centre.

Rule. Multiply the number in the table of Multiplicands, by the square of tho depth in inches, and four times the breadth; divide that product by the length in inches, and the quotient will be the weight.

EXAMPLE. I.

What weight will break a beam of English Oak 7 inches broad, 9 inches deep, and 30 feet between the props?

$$\frac{1426 \times 81 \times 28}{360} = 8983\tfrac{288}{360} \text{ libs.}$$

EXAMPLE II.

A beam of Beech, 7 inches deep, 4 inches broad, and 10 feet long, supports a weight of 4 tons, what additional weight will require to be added to break the beam?

$$\frac{1556 \times 49 \times 16}{120} = 10332 - 8960 = 1372 \text{ libs.}$$

EXAMPLE III.

What will be the dimensions of a Fir Beam 30 feet long between the props, to support a weight of 6000 libs.

$$\frac{6000 \times 360}{1100} = 1963.\dot{6}\dot{3} \text{ the square of the depth,}$$

and 4 times the breadth.

Suppose the breadth 6 inches.

$$6 \times 4 = 24 \quad \frac{1963.\dot{6}\dot{3}}{24} = 81 \text{ square of the depth,}$$

and $\sqrt{81.81} = 9.5 =$ depth.

Suppose the depth 10 inches.

$$10^2 = 100 \quad \frac{1963.\dot{6}\dot{3}}{100 \times 4} = 4.75 \text{ breadth.}$$

When the beam is uniformly loaded throughout its length, the result must be doubled, *i. e.* it will support double the weight.

When the beam is fixed at both ends and loaded in the middle, one half of the result must be added; and if the weight is laid uniformly along its length, the result must be tripled.

These problems are taken from Barlow's Essay, as before quoted: he, however, gives a second Rule to each of the Problems, in which the angle of deflection is considered. These rules give higher results than those here stated; but for practice the first rule is the best, being more simple and safe.

It is considered that two-thirds of the result is sufficient to lay upon a beam for a permanent load.

STIFFNESS OF BEAMS, BARS, &c.

It is frequently far more important in practice to determine the weight a Beam will bear without bending, than to determine its absolute strength; the following problems have therefore been added.

PRACTICAL PROBLEMS *for the Stiffness of Timber.**

TABLE OF CONSTANT QUANTITIES.

English Oak	.01
Canadian Oak	.009
Ash	.0113
Beech	.01277
Elm	.0126
Pitch Pine	.0166
White Pine	.0121

PROBLEM I.

To find the dimensions of a horizontal Beam, supported at both ends, to resist flexure, when loaded with a given weight.

CASE I.

When the breadth is given.

RULE. Multiply the square of the length in feet by the weight in pounds, and the product by the constant quantity from the table. Divide the product by the breadth in inches, the cube root of the product will be the depth in inches.

EXAMPLE.

What should be the depth of a horizontal beam of White Pine, supported at the ends, 3 inches in breadth and 30 feet in length, to support without bending, a weight of 1000 libs;

$\frac{30\times30\times.0121}{3}=3630$ whose cube root is 15.4 nearly.

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* See Tredgold's "*Elementary Principles of Carpentry.*"

CASE II.

When the depth is given.

Rule. Multiply the square of the length in feet by the weight in pounds, and the product by the constant quantity from the table. Divide the last product by the cube of the depth in inches, and the quotient will be the required breadth.

EXAMPLE.

What should be the breadth of a horizontal Beam of White Pine, supported at the ends, 30 feet in length, and 15 inches in depth, to support a weight of 1000 libs without flexure.

$$\frac{30 \times 30 \times .0121}{15 \times 15 \times 15} = 3.227$$

CASE III.

When the ratio between the breadth and depth is given.

Rule. Multiply the weight in pounds by the constant quantity from the table: divide the product by the ratio of the breadth to the depth, and extract the square root. Multiply this root by the length in feet, and extract the square root a second time, which will be the depth in inches required. The breadth is equal to the depth multiplied by the fraction expressing the ratio.

EXAMPLE.

What should be the dimensions of a horizontal Beam of White Pine, supported at both ends, to bear a weight of 1000 libs, when the breadth is one third of the depth, and the length 30 feet.

$\frac{1000 \times .0121}{\frac{1}{3}} = 36.500$ whose square root a little exceeds 6.

$6 \times 30 = 180$ whose square root is 13.5, nearly.

One third of 13.5 is 4.5.

The Beam therefore should be 4½ inches broad, by 13½ deep.*

PROBLEM II.

To find the load that can be borne by a beam whose dimensions are given, without bending.

RULE. Multiply the breadth by the cube of the depth in inches, and divide the product by the square of the length in feet, multiplied by the con-

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* The algebraic formulæ are

$$1.\ d = \left(\frac{a L^2 w}{b}\right)^{\frac{1}{3}}$$

$$2.\ b = \frac{a L^2 w}{d^3}$$

$$3.\ d^2 = L \times \left(\frac{a w}{r}\right)^{\frac{1}{2}}$$

stant quantity from the tables.* The quotient will be the weight in pounds.

EXAMPLE.

Required the weight that a horizontal Beam of White Pine, 3 inches broad, 15 inches deep, and 30 feet long, will bear without bending.

$$\frac{3\times15\times15\times15}{30\times30\times.0121}=929 \text{ libs.}$$

PRACTICAL PROBLEMS *For the Transverse Strength of Cast Iron Beams.*†

PROBLEM I.

To find the breadth of a uniform Cast Iron Beam to bear a given weight in the middle.

RULE 1. Multiply the length of bearing in feet, or the length between the supports, by the weight to be supported in libs; and divide this product by 350 times the square of the depth in inches; the quotient will be the breadth in inches required.

RULE 2. Multiply the length of bearing in feet, by the weight to be supported in libs, and divide

---

* The algebraic formula is

$$W=\frac{b\,d^2}{a\,L^2}$$

† See Tredgold's Practical Essay on the Strength of Cast Iron, p. 80.

this product by 850 times the breadth in inches; and the square root of the quotient will be the depth in inches.

When no particular breadth or depth is determined by the nature of the situation for which the beam is intended, it will be found sometimes convenient to assign some proportion; as, for example, let the breadth be the n^{th} part of the depth, n representing any number at will. Then the rule is as follows:—

Multiply n times the length in feet, by the weight in libs; divide this product by 850, and the cube root of the quotient will be the depth required; and the breadth will be the n^{th} part of the depth.

Note. It may be remarked here, that the Rules are the same for inclined as for horizontal beams, when the horizontal distance between the supports is taken for the length of bearing.

EXAMPLE I.

What is the breadth of a Beam 20 feet long, 15 inches deep, and to be loaded with 13 tons?

13 tons = 29120 libs.

$$\frac{29120 \times 20}{15^2 \times 850} = 3.045 \text{ inches broad.}$$

EXAMPLE II.

What is the depth of a beam 20 feet long, 3 inches broad, and to support a weight of 13 tons?

$\frac{29120 \times 20}{850 \times 3} = 225$, the square root of which is $= 15$ inches, the depth required.

EXAMPLE III.

What are the cross sectional dimensions of a Beam 30 feet long, and of sufficient strength to support a weight of 10 tons; the depth being twice the breadth?——n will therefore be $= 2$

10 tons $=$ 22400 libs. Length $= 30$ $30 \times 2 = 60$

$\frac{22400 \times 60}{850} = 1581$, the cube root of which is nearly $11\frac{1}{2}$, which is equal to the depth in inches: the breadth is the half of the depth $= 5\frac{3}{4}$ inches.

When the dimensions of the Beam are given, to find the weight.

RULE. Multiply 850 times the square of the depth by the breadth, and divide by the length in feet; the quotient will be the weight in libs. which the beam will carry.

EXAMPLE IV.

What weight will a Beam of the following dimensions bear?

Length,............20 feet.
Breadth,............$2\frac{1}{4}$ inches.
Depth,............12 do.

$$\frac{12^2 \times 850 \times 2.25}{20} = 13770 \text{ libs weight.}$$

	Tons.	Cwt.	Qrs.	Libs.
Or	6	2	3	22

PROBLEM II.

To find the breadth, when the load is not in the middle between the supports.

Rule. Multiply the short length by the long length, and four times this product divided by the whole length between the supports, will give the effective leverage of the load in feet; this quotient being used instead of the length, in any of the Rules in the foregoing Problem, the breadth and depth will be found by them.

EXAMPLE.

What are the cross sectional dimensions of a Beam 12 feet long, supporting a weight of 15 tons, 3 feet from the one end, when the breadth is a fourth of the depth?

$$\frac{3 \times 9 \times 4}{12} = 9 \quad 9 \times 4 = 36 \quad 15 \text{ tons} = 33600 \text{ libs.}$$

$$\frac{33600 \times 36}{850} = 1423,$$ the cube root of which is $= 11\frac{1}{4}$, the depth: the breadth will be $\frac{11\frac{1}{4}}{4} = 2\frac{13}{16}$

PROBLEM III.

To find the breadth when the load is uniformly distributed over the length of the beam.

Rule. The same Rules apply as in Prob. 1, only the divisor is changed from 850 to 1700, *i. e.* when the load is uniformly distributed over the length of the beam, it supports double the weight than when the load is laid on the middle.

Note. Examples in Problem 1. apply to this Problem, only changing the divisors, or halving the quotients.

PROBLEM IV.

To find the dimensions, when a beam is fixed at one end and loaded at the other, or when it is supported at the middle and loaded at both ends.

Rule. Take the horizontal length of the projection of the beam, when fixed at one end, for the length, and apply the Rules in Prob. 1, only using the divisor 212 instead of 850.

When the beam is supported any where between the two ends, multiply the length from the prop by the weight hung at the end, and apply the remainder of the Rule as in Prob. 1, only using 212 for 850.

When the load is uniformly distributed over the length of the projection, employ 425 instead of 212 as a divisor.

Note. The rules of this Problem apply to the teeth of wheels, the length being the length of the teeth, and the depth, the thickness of the teeth.

Example to this Note.

Let the greatest power acting at the pitch line of the wheel be 6000 libs, and the thickness of the teeth 1½ inch, the length of the teeth being ¼ foot; What is the breadth of the teeth?

$\frac{6000 \times .25}{212 \times \overline{1.5}^2} = \frac{1500}{477} = 3.14$ inches the breadth; but to allow for wearing by friction, this quotient is doubled, or 6¼ inches = the breadth of the teeth, or face of the wheel.

PROBLEM V.

To find the diameter of a solid cylinder to support a given weight in the middle—between the middle and the end—and when the weight is uniformly distributed over the length—also when fixed at one end.

When the weight is placed in the middle.

Rule. Multiply the weight in libs by the length in feet; divide this product by 500, and the cube root of the quotient will be the diameter in inches.

When the weight is between the middle and the end.

Rule. Multiply the short end by the long end; then multiply that product by 4 times the weight in libs. Divide this product by 500 times the length in feet, and the cube root of the quotient will be the diameter in inches.

When the load is uniformly distributed over the length.

Rule. Multiply the length in feet by the weight in libs, and one-tenth of the cube root of the product will be the diameter in inches.

When fixed at one end, and the load applied at the other.

Rule. Multiply the length of projection in feet by the weight in libs, and the 5th part of the cube root of this product will be the diameter in inches.

The Rules for the deflection of Beams and Bars are here omitted, being considered, that, in most of practical cases, the deflection is of little importance; however, when it is of importance, reference to Barlow's Essay on the strength of Timber, and Tredgold's Essay on the strength of Iron, will satisfy all inquiries. These books ought to be in the possession of every Mechanic, as they give the most comprehensive and most correct data for the strength of materials of any that have yet appeared.

STRENGTH OF THE JOURNALS OF SHAFTS.

LATERAL STRENGTH.

The Rules in Problem 5. of last article, can be here applied. Mr. Robertson Buchanan, in his Essay on the Strength of Shafts, uses the following Rule, which is simple enough, and easy to be remembered; but the above-mentioned Rules are the most correct, and ought to be used on all occasions.

Mr. Buchanan's Rule is,—"The cube root of the weight in cwts. is nearly equal to the diameter of the Journal."—"*Nearly equal,*—being prudent to make the Journal a little more than less, and to make a due allowance for wearing."

EXAMPLE.

What is the diameter of the Journal of a Water Wheel Shaft, 13 feet long, the weight of the Wheel being 15 tons?

By Mr. B's Rule. $\sqrt[3]{15\times 20}=6.7$ or 7 inches diam$^{r.}$

By Mr. Tredgold's Rule.

Weight in the middle. $\left\{ \dfrac{33600 \times 13}{500} = 873 \quad \sqrt[3]{873} = 9\tfrac{1}{2} \right.$ inches diam$^{r.}$

Weight equally distributed. $\left\{ 33600\times 13=436800 \quad \sqrt[3]{\dfrac{436800}{10}} = 7.65 \right.$ In.

TO RESIST TORSION OR TWISTING.

It is obvious that the strength of revolving Shafts are directly as the cubes of their diameters and revolutions; and inversely, as the resistance they have to overcome.

Mr. Robertson Buchanan, in his Essay on the strength of Shafts, gives the following data, deduced from several experiments, viz. That the Fly Wheel Shaft of a 50 horse power engine, at 50 revolutions per minute, requires to be 7½ inches diameter, and therefore, the cube of this diameter, which is = 421.875, serves as a multiplier to all other Shafts in the same proportion: and taking this as a standard, he gives the following multipliers, viz.

For the Shaft of a Steam Engine, Water Wheel, or any Shaft connected with a first power, - - - - -	400
For Shafts in inside of Mills, to drive smaller Machinery, or connected with the Shafts above, - - - - - - -	200
For the small Shafts of a Mill or Machinery,	100

From the foregoing, the following Rule is derived, viz.

The number of horses' power a shaft is equal to, is directly as the cube of the diameter and number of revolutions; and inversely, as the above Multipliers.

Note. Shafts here are understood as the Journals of Shafts, the bodies of Shafts being generally made square.

EXAMPLE I.

When the Fly Wheel Shaft of a 45 horse power Steam Engine makes 90 revolutions per minute, what is the diameter of the Journal?

$\frac{45 \times 400}{90} = 200 \quad \sqrt[3]{200} = 5\frac{8}{10}$ inches diameter.

EXAMPLE II.

The velocity of a Shaft is 80 revolutions per minute, and its diameter is 3 inches: What is its power?

$\frac{3^3 \times 80}{400} = 5.4$ horse power.

EXAMPLE III.

What will be the diameter of the Shaft in the first Example, when used as a Shaft of the second Multiplier?*

$\frac{5.8}{1.25} = 4.64$, or $\sqrt[3]{\frac{45 \times 200}{90}} = 4\frac{6}{10}$ inches diameter.

The following is a Table of the diameters of Shafts, being the First Movers, or having 400 for their multipliers.

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* The diameters of the second movers will be found by dividing the numbers in the table by 1.25, and the diameters of the third movers, by dividing the numbers by 1.56.

TABLE.

DIAMETERS OF THE JOURNALS OF FIRST MOVERS.

Horses' Power.	REVOLUTIONS.									
	10	15	20	25	30	35	40	45	50	55
4	5.5	4.8	4.5	4.	3.7	3.8	3.5	3.3	3.2	3.1
5	5.9	5.1	4.7	4.4	4.1	3.9	3.7	3.6	3.5	3.3
6	6.3	5.5	5.	4.6	4.4	4.1	4.	3.8	3.7	3.6
7	6.6	5.8	5.2	4.9	4.6	4.4	4.2	4.	3.9	3.7
8	6.9	6.	5.5	5.1	4.8	4.6	4.4	4.2	4.1	4.
9	7.2	6.3	5.7	5.5	5.	4.8	4.5	4.4	4.2	4.1
10	7.4	6.6	5.9	5.6	5.2	4.9	4.7	4.6	4.4	4.2
12	7.9	6.9	6.3	5.8	5.6	5.4	5.2	5.	4.8	4.6
14	8.3	7.2	6.7	6.2	5.9	5.6	5.4	5.2	5.	4.7
16	8.7	7.6	7.1	6.6	6.1	5.8	5.6	5.4	5.2	5.
18	9.	7.9	7.5	7.	6.6	6.2	5.8	5.6	5.4	5.2
20	9.3	8.1	7.4	7.2	6.6	6.4	5.9	5.7	5.6	5.4
25	10.	8.5	8.	7.4	7.1	6.8	6.3	6.	5.9	5.6
30	10.7	9.3	8.4	7.9	7.4	7.1	6.9	6.7	6.5	6.3
35	11.4	9.8	8.9	8.4	7.9	7.4	7.1	6.9	6.6	6.5
40	11.7	10.5	9.3	8.8	8.3	7.8	7.4	7.2	6.9	6.7
45	12.	10.6	9.7	9.2	8.7	8.1	7.6	7.4	7.	6.8
50	12.6	11.	10.	9.3	9.	8.5	[illegible]	7.8	7.4	7.3
55	13.4	11.4	10.4	9.8	9.1	8.8	8.4	8.	7.5	7.4
60	13.6	12.	10.8	10.	9.3	9.	8.6	8.2	7.7	7.6
INCHES DIAMETER.										

TABLE CONTINUED.

Horses' Power.	REVOLUTIONS.									
	60	65	70	75	80	85	90	95	100	105
4	3.	2.9	2.9	2.8	2.7	2.7	2.6	2.6	2.6	2.5
5	3.3	3.2	3.1	3.	3.	2.9	2.9	2.8	2.8	2.7
6	3.5	3.5	3.4	3.3	3.2	3.2	3.	3.	2.9	2.9
7	3.6	3.6	3.5	3.4	3.4	3.3	3.3	3.2	3.1	3.1
8	3.9	3.8	3.7	3.6	3.5	3.5	3.4	3.4	3.3	3.2
9	4.	3.8	3.7	3.7	3.6	3.6	3.5	3.5	3.4	3.3
10	4.1	4.	3.9	3.8	3.7	3.7	3.6	3.6	3.5	3.4
12	4.4	4.3	4.2	4.1	4.	3.9	3.8	3.8	3.7	3.6
14	4.5	4.4	4.4	4.3	4.2	4.1	4.	4.	3.9	3.8
16	4.8	4.7	4.6	4.5	4.4	4.4	4.3	4.2	4.1	4.
18	5.	4.9	4.8	4.7	4.6	4.5	4.4	4.3	4.2	4.2
20	5.2	5.1	5.	4.8	4.6	4.6	4.5	4.5	4.4	4.4
25	5.5	5.4	5.3	5.2	5.1	4.9	4.8	4.7	4.6	4.6
30	5.9	5.8	5.7	5.6	5.5	5.3	5.2	5.1	5.	4.9
35	6.3	6.1	5.9	5.7	5.6	5.5	5.4	5.3	5.2	5.2
40	6.6	6.4	6.2	6.	5.9	5.8	5.7	5.6	5.6	5.5
45	6.7	6.5	6.4	6.2	6.1	6.	5.9	5.8	5.7	5.6
50	7.2	6.9	6.8	6.6	6.5	6.4	6.2	6.	5.9	5.8
55	7.3	7.2	7.	6.7	6.6	6.5	6.3	6.2	6.1	6.
60	7.4	7.3	7.2	6.9	6.8	6.8	6.7	6.6	6.4	6.2
	INCHES DIAMETER.									

It is a well known fact, that a cast iron rod will sustain more torsional pressure, than a malleable iron rod of the same dimensions.—That is, a malleable iron rod will be twisted by a less weight than what is required to wrench a cast iron rod of the same dimensions.

When the strength of malleable iron is less than that of cast iron to resist torsion; it is stronger than cast iron to resist lateral pressure, and that strength is in proportion as 9 is to 14.

From the foregoing, it is easy for the Mill-wright to make his Shafts of the iron best suited to overcome the resistance to which they will be subject, and the proportion of the diameters of their Journals, according to the iron of which they are made: for example; What will be the diameter of a malleable iron Journal, to sustain an equal weight with a cast iron Journal of 7 inches diameter?

$$7^3 = 343$$

$14 : 343 :: 9 : 220\frac{1}{2}$ now $\sqrt[3]{220.5} = 6.04$ inches diameter.

STRENGTH OF WHEELS.

THE arms of Wheels are as levers fixed at one end and loaded at the other, and consequently the greatest strain is upon the end of the arm next the axle; for that reason all arms of wheels should be strongest at that part, and tapering towards the rim.

The Rule for the breadth and thickness of arms, according to their length and number in the wheel, is as follows: (*See Tredgold's Essay, page* 114.) Multiply the power or weight acting at the end of the arm by the cube of its length; the product of which, divided by 2656 times the number of arms multiplied by the deflection, will give the breadth and cube of the depth.

EXAMPLE.

Suppose the force acting at the circumference of a spur wheel to be 1600 libs, the radius of wheel 6 feet, and number of arms 8, and let the deflection not exceed $\frac{1}{10}$ of an inch.

$$\frac{1600 \times 6^3}{2656 \times 8 \times .1} = 163 = \text{breadth and cube of the depth.}$$

Let the breadth be 2.5 inches, therefore $\frac{163}{2.5} = 65.2$, which is equal to the cube of the depth: now the cube root of 65.2 is nearly 4.03 inches: this, conse-

quently, is the depth, or dimension, of each arm in the direction of the force.

Note. When the depth at the rim is intended to be half that of the axes, use 1640 as a divisor instead of 2656.

The teeth are as beams, or cantilevers, fixed at one end and loaded at the other, the Rule applying direct to them (*See Tredgold's Essay, Art.* 121) where the length of the beam is the length of the teeth, and the depth the thickness of the teeth. For the better explanation of the Rule the following Example is given.

EXAMPLE.

The greatest power acting at the pitch line of the wheel is 6000 libs, and the thickness of the teeth 1½ inch, the length of the teeth being 0.25 feet; it is required to determine the breadth of the teeth.

$\frac{6000 \times 0.25}{212 \times \overline{1.5}^2} = \frac{1500}{477} = 3.2$ inches the breadth required.

In order that the teeth may be capable of offering a sufficient resistance after being worn by friction, the breadth thus found should be doubled; therefore, in the above Example, the breadth should be 6.4, or say 6½ inches.

Mr. Carmichael* gives the following data gleaned from experiments, which are therefore valuable, and of much use to the practical mechanic.

* See Robertson Buchanan on the Teeth of Wheels.

RULE. Multiply the breadth of the teeth by the square of the thickness, and divide the product by the length; the quotient will be the proportional strength in horses' power, with a velocity of 2.27 feet per second.

EXAMPLE.

What is the power of a wheel, the teeth of which are 6 inches broad, 1.5 inch thick, and 1.8 inch long, and revolving at the velocity of 3 feet per second?

$$\frac{\overline{1.5}^2 \times 6}{1.8} = \frac{13.5}{1.8} = 7.5 \text{ strength at } 2.27 \text{ feet per sec.}$$

$$\text{then } 2.27 : 7.5 :: 3 = \frac{7.5 \times 3}{2.27} = 9.91 \text{ horse-power.}$$

RULE. The pitch is found by multiplying the thickness by 2.1, and the length is found by multiplying the thickness by 1.2.

EXAMPLE.

The thickness being 2 inches, what is the pitch and length?

$$2 \times 2.1 = 4.2 \text{ Pitch.}$$
$$2 \times 1.2 = 2.4 \text{ Length.}$$

Note. The breadth of the teeth, as commonly executed by the best Masters, seems to be from about twice to thrice the pitch.

TABLE.

Pitch in Inches.	Thick-ness in Inches.	Breadth in Inches.	Length in Inches.	Horses' Power at 2.27 feet per Second.	H.P. at 3 feet per Second.	H. P. at 6 feet per Second.	H. P. at 11 feet per Second.
4.2	2.	8.	2.40	13.33	17.61	35.23	64.6
3.99	1.9	7.6	2.28	13.03	15.90	31.80	58.30
3.78	1.8	7.2	2.16	10.80	14.27	28.54	52.32
3.57	1.7	6.8	2.04	9.63	12.72	25.54	46.68
3.36	1.6	6.4	1.92	8.53	11.27	22.54	41.32
3.15	1.5	6.	1.80	7.50	9.91	19.82	36.33
2.94	1.4	5.6	1.68	6.53	8.63	17.26	31.64
2.73	1.3	5.2	1.56	5.63	7.44	14.88	27.28
2.52	1.2	4.8	1.44	4.80	6.34	12.68	23.24
2.31	1.1	4.4	1.32	4.03	5.32	10.64	19.54
2.10	1.	4.	1.20	3.33	4.40	8.81	16.15
1.89	.9	3.6	1.08	2.70	3.57	7.14	13.09
1.68	.8	3.2	.96	2.13	2.81	5.62	10.33
1.47	.7	2.8	.84	1.63	2.15	4.30	7.88
1.26	.6	2.4	.72	1.20	1.59	3.18	5.83
1.05	.5	2.	.60	.83	1.10	2.20	4.03

VELOCITY OF WHEELS.

Wheels are for conveying motion to the different parts of a machine, at the same, or at a greater or less velocity, as may be required.—When two wheels are in motion their teeth act on one another alternately, and consequently, if one of these wheels has 40 teeth, and the other 20 teeth, the one with 20 will turn twice upon its axis for one revolution of the wheel with 40 teeth.—From this the Rule is taken, which is,—As the velocity required is to the number of teeth in the driver, so is the velocity of the driver to the number of teeth in the driven.

Note. To find the proportion that the velocities of the wheels in a train should bear to one another, subtract the less velocity from the greater, and divide the remainder by the number of one less than the wheels in the train; the quotient will be the number rising in Arithmetical progression, from the least to the greatest velocity of the train of wheels.

EXAMPLE I.

What is the number of teeth in each of three wheels to produce 17 revolutions per minute, the driver having 107 teeth; and making three revolutions per minute?

$\dfrac{17-3=14}{3-1=\ \ 2}=7$, therefore 3 10 17 are the velocities of the three wheels.

By the Rule $\begin{cases} 10 : 107 :: 3 : 32 = \dfrac{107\times 3}{10} = 32 \text{ teeth.} \\ 17 : 32 :: 10 : 19 = \dfrac{32\times 10}{17} = 19 \text{ teeth.} \end{cases}$

EXAMPLE II.

What is the number of teeth in each of 7 wheels, to produce 1 revolution per minute, the driver having 25 teeth, and making 56 revolutions per minute?

$\dfrac{56-1=55}{7-1=\ \ 6}=9$, therefore 56 46 37 28 19 10 1, are the progressional velocities.

46	:	25	: :	56	:	30	Teeth.
37	:	30	: :	46	:	37	——
28	:	37	: :	37	:	49	——
19	:	49	: :	28	:	72	——
10	:	72	: :	19	:	137	——
1	:	137	: :	10	:	1370	——

It will be observed that the last wheel, in the foregoing Example, is of a size too great for application;

to obviate this difficulty, which frequently arises in this kind of training, wheels and pinions are used, which give a great command of velocity.—Suppose the velocities of last Example, and the train only of 2 wheels and 2 pinions.

$$\frac{56-1=55}{4-1=3}=18,$$ therefore 56 19 1, are the progressional velocities.

19 : 25 : : 56 : 74 = teeth in the wheel driven by the first driver, and 1 : 10 : : 19 : 190=teeth, in the second driven wheel, 10 teeth being in the driving pinion.

25 drivers	74 driven.
10 ———	190 ———

The following is a Table of the Radii of Wheels, from ten to three hundred teeth, the pitch being 2 inches.

The radius for any other pitch may be found by the following analogy:—As two Inches is to the Radius in the Table, so is the new Pitch to the new Radius.

TABLE.

No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.
10	3.236	46	14.654	82	26.108	118	37.565
11	3.549	47	14.972	83	26.426	119	37.883
12	3.864	48	15.290	84	26.741	120	38.202
13	4.179	49	15.608	85	27.063	121	38.520
14	4.494	50	15.926	86	27.381	122	38.838
15	4.810	51	16.244	87	27.699	123	39.156
16	5.126	52	16.562	88	28.017	124	39.475
17	5.442	53	16.880	89	28.336	125	39.793
18	5.759	54	17.198	90	28.654	126	40.111
19	6.076	55	17.517	91	28.972	127	40.429
20	6.392	56	17.835	92	29.290	128	40.748
21	6.710	57	18.153	93	29.608	129	41.066
22	7.027	58	18.471	94	29.927	130	41.384
23	7.344	59	18.789	95	30.245	131	41.703
24	7.661	60	19.107	96	30.563	132	42.021
25	7.979	61	19.425	97	30.881	133	42.339
26	8.296	62	19.744	98	31.200	134	42.657
27	8.614	63	20.062	99	31.518	135	42.976
28	8.931	64	20.380	100	31.836	136	43.294
29	9.249	65	20.698	101	32.155	137	43.612
30	9.567	66	21.016	102	32.473	138	43.931
31	9.885	67	21.335	103	32.791	139	44.249
32	10.202	68	21.653	104	33.109	140	44.567
33	10.520	69	21.971	105	33.427	141	44.885
34	10.838	70	22.289	106	33.746	142	45.204
35	11.156	71	22.607	107	34.064	143	45.522
36	11.474	72	22.926	108	34.382	144	45.840
37	11.792	73	23.244	109	34.700	145	46.158
38	12.110	74	23.562	110	35.018	146	46.477
39	12.428	75	23.880	111	35.337	147	46.795
40	12.746	76	24.198	112	35.655	148	47.113
41	13.064	77	24.517	113	35.974	149	47.432
42	13.382	78	24.835	114	36.292	150	47.750
43	13.700	79	25.153	115	36.611	151	48.068
44	14.018	80	25.471	116	36.929	152	48.387
45	14.336	81	25.790	117	37.247	153	48.705

TABLE CONTINUED.

No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.	No. of Teeth	Radius in Inches.
154	49.023	191	60.800	228	72.577	265	84.354
155	49.341	192	61.118	229	72.895	266	84.673
156	49.660	193	61.436	230	73.214	267	84.991
157	49.978	194	61.755	231	73.532	268	85.309
158	50.296	195	62.073	232	73.850	269	85.627
159	50.615	196	62.392	233	74.168	270	85.946
160	50.933	197	62.710	234	74.487	271	86.265
161	51.251	198	63.028	235	74.805	272	86.582
162	51.569	199	63.346	236	75.123	273	86.900
163	51.888	200	63.665	237	75.441	274	87.219
164	52.206	201	63.983	238	75.760	275	87.537
165	52.524	202	64.301	239	76.078	276	87.855
166	52.843	203	64.620	240	76.397	277	88.174
167	53.161	204	64.938	241	76.715	278	88.462
168	53.479	205	65.256	242	77.033	279	88.810
169	53.798	206	65.574	243	77.351	280	89.129
170	54.116	207	65.893	244	77.670	281	89.447
171	54.434	208	66.211	245	77.988	282	89.765
172	54.752	209	66.529	246	78.306	283	90.084
173	55.071	210	66.848	247	78.625	284	90.402
174	55.389	211	67.166	248	78.943	285	90.720
175	55.707	212	67.484	249	79.261	286	91.038
176	55.026	213	67.803	250	79.580	287	91.357
177	55.344	214	68.121	251	79.898	288	91.675
178	56.662	215	68.439	252	80.216	289	91.993
179	56.980	216	68.757	253	80.534	290	92.312
180	57.299	217	69.075	254	80.853	291	92.630
181	57.617	218	69.394	255	81.171	292	92.948
182	57.935	219	69.712	256	81.489	293	93.267
183	58.253	220	70.031	257	81.808	294	93.585
184	58.572	221	70.349	258	82.126	295	93.903
185	58.890	222	70.667	259	82.444	296	94.222
186	59.209	223	70.985	260	82.763	297	94.540
187	59.527	224	71.304	261	83.081	298	94.858
188	59.845	225	71.622	262	83.399	299	95.177
189	60.163	226	71.941	263	83.717	300	95.495
190	60.482	227	72.258	264	84.038		

CENTRE OF GRAVITY.

The centre of Gravity of a body is that point, which, if sustained, the body remains at rest; the particles of which it is composed being equipoised, and having their weights collected, as it were, into that point.

Bodies are reciprocal to each other, as their distances from the centre of gravity.—Suppose a rod 11 inches long, with a weight of 2 libs hung at the one end, and a weight of 20 libs hung at the other end, the centre of gravity, or the point on which this rod so loaded, will balance itself, is just 1 inch from the greater weight, and 10 inches from the less, because, $20 \times 1 = 20$, and $2 \times 10 = 20$, therefore, their weights are inversely as their distances from the centre of gravity.—Hence, the method to find the common centre of gravity of any number of bodies, is, first find the centre between two bodies, then the centre between that centre and a third body, and so on for a fourth, fifth, &c.; the last centre found being the common centre of all the bodies.

From the foregoing it will easily be conceived, that if a homogeneous beam is balanced upon a

point, that point will be the centre of gravity, and also the centre of the beam; but suppose the beam 10 feet long, each foot weighing 8 libs, and a weight of 90 libs suspended from the one end, at what point of the beam will the centre of gravity be?

10 feet, length of beam.—8 libs each foot in length.
90 libs weight suspended.

$$\frac{8\times10+2\times90}{8\times10+90}\times\frac{10}{2}=\frac{260}{170}\times5=7.65$$

$+2.35=10$ feet length of beam, that is, the centre of gravity is 2.35 feet from the end at which the weight of 90 libs is suspended, and will be 7.65 feet from the other end.

Suppose another homogeneous beam, 12 feet long, with a weight of 100 libs fixed at one end, it is found that the whole is in equilibrio when the beam is suspended 2 feet from the end next the weight; what is the weight of the beam?

100 libs weight suspended.
2 feet distance from the weight.
10 feet distance from the other end.

$\frac{2\times100\times2}{100-4}=\frac{400}{96}=4.166$ libs the weight of 1 foot of beam, and $4.166\times12=49.992$ libs, the weight of the beam.

It is well known to every practical Mechanic, that there are no homogeneous beams or bars:—that it is impossible to find the weight of a foot of length, in a piece of wood, iron, stone, &c. and that the exact

centre of gravity of such materials cannot be found by any known theorem. To obviate this difficulty, and ascertain the true centre of gravity, the beams, bars, &c. are balanced over a prop; but there are many large unwieldy bodies that cannot be thus treated, and for this reason the following data are given, which ascertain nearly the centre required; the data being taken, which are nearest the form and distribution of weight over the body, of which the centre of gravity is required.

1. The centre of gravity of a triangle is in the straight line, drawn from any angle to the bisection of the opposite side, at the distance of $\frac{2}{3}$ of that line from the angle.

This rule is also true with regard to a pyramid of any number of sides; also to a cone.

2. The centre of gravity of a segment of a circle, is in the radius which bisects it; and its distance from the centre of the circle, is $\frac{1}{12}$ of the cube of its chord divided by the area of the segment.

3. The centre of gravity of a sector of a circle is in the radius which bisects it; and its distance from the centre of the circle, is a fourth proportional to the arc, its chord, and $\frac{2}{3}$ of the radius.

For further information in this article, *see Hutton's Mathematics, Banks on Mills, Venturoli's Mechanics by Cresswell, &c.*

CENTRE OF PERCUSSION.

The centre of Oscillation or Percussion, is the point in a body revolving round a fixed axis, so taken, that when it is stopped by any force, the whole motion, and tendency to motion, of the revolving body, is stopped at the same time.

It is also that point of a revolving body, which would strike any obstacle with the greatest effect; and from this property it has received the name of percussion.

The centres of oscillation and percussion are generally treated separately; but the two centres are in the same point, and therefore their properties are the same.

As in bodies at rest, the whole weight may be considered as collected in the centre of gravity; so in bodies in motion, the whole force may be considered as concentrated in the centre of percussion:—therefore, the weight of the rod multiplied by the distance of the centre of gravity from the point of suspension, will be equal to the force of the rod divided by the distance of the centre of percussion from the same point. For example, the length of a rod being 20 feet, and the weight of a foot in length equal to 100 oz.; also, a weight or ball fixed

at under end weighing 1000 oz.; at what point of the rod from the point of suspension will be the centre of percussion?*

The weight of the rod is $20 \times 100 = 2000$ oz. which multiplied by half its length $2000 \times 10 = 20000$, gives the momentum of the rod. The weight of the ball = 1000 oz. multiplied by the length of rod $= 1000 \times 20$, gives the momentum of the ball. Now the weight of the rod multiplied by the square of the length; and divided by

$$3 = \frac{2000 \times 20^2}{3} = 266666, = \text{the force of the rod,}$$

and the weight of the ball multiplied by the square of the length of the rod, $1000 \times 20^2 = 400000$, is the force of the ball:—therefore the centre of

$$\text{percussion} = \frac{266666 + 400000}{20000 + 20000} = \frac{666666}{40000} = 16.66 \text{ feet.}$$

For another example; suppose a rod 12 feet long, and 2 lbs. each foot in length, with 2 balls of 3 lbs. each, one fixed 6 feet from the point of suspension, and the other at the end of the rod; what is the distance between the points of suspension and percussion?

$12 \times 2 \times 6$	=	144	momentum	of rod.
3×6	=	18	do.	of 1st. ball.
3×12	=	36	do.	of 2d. do.
		198		

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*a=20 feet long.
b=100 oz. wt. of a foot in length.
c=1000 do. fixed at end.

$$\frac{\frac{1}{2}ab \times a^2 + ca^2}{\frac{1}{2}ab \times a + ac} = \text{centre of percussion.}$$

$$\frac{24\times144}{3}=1152, \text{ force of rod.}$$

$$3\times\ 36=\ 108 \text{ do. of 1st. ball.}$$

$$3\times144=\ 432 \text{ do. of 2d. ball.}$$

$$1692$$

therefore the centre of percussion $=\frac{1692}{198}=8.5\dot{4}\dot{5}$ feet from the point of suspension.

As the centre of percussion is the same with the centre of gravity in the non-application to practical purposes, the following is the easiest and simplest mode of finding it in any beam, bar, &c.

Suspend the body very freely by a fixed point, and make it vibrate in small arcs, counting the number of vibrations it makes in any time, as a minute, and let the number of vibrations made in a minute be called n; then shall the distance of the centre of oscillation from the point of suspension be $\frac{140850}{n^2}=$inches.—For the length of the pendulum vibrating seconds, or 60 times in a minute, being $39\frac{1}{8}$ inches; and the lengths of the pendulums being reciprocally as the square of the number of vibrations made in the same time:—therefore, $n^2 : 60^2 :: 39\frac{1}{8} : \frac{60^2\times39\frac{1}{8}}{n^2}=\frac{140850}{n^2}$ being the length of the pendulum which vibrates n times in a minute, or the distance of the centre of oscillation below the axis of motion.

There are many situations in which bodies are placed, that prevent the application of the above

rule; and for this reason the following data are given, which will be found useful when the bodies and the forms here given correspond.

1. If the body is a heavy straight line of uniform density, and is suspended by one extremity, the distance of its centre of percussion is $\frac{2}{3}$ds of its length.

2. In a slender rod of a cylindrical or prismatic shape, whose breadth is very small compared with its length, has the distance of its centre of percussion nearly $\frac{2}{3}$ds of its length from the axis of suspension.

If these rods were formed so that all the points of their transverse sections were equidistant from the axis of suspension, the distance of the centre of percussion would be exactly $\frac{2}{3}$ds of their length.

3. In an Isosceles Triangle, suspended by its apex, and vibrating in a plane perpendicular to itself, the distance of the centre of percussion is $\frac{3}{4}$ths of its altitude. A line or rod, whose density varies as the distance from its extremity, or the point of suspension; *also a fly-wheel, or wheels in general,* is in precisely the same predicament as the Isosceles Triangle; *i. e.* the centre of percussion is distant from the centre of suspension $\frac{3}{4}$ths of its length.

4. In a very slender cone or pyramid, vibrating about its apex, the distance of its centre of percussion is nearly $\frac{4}{5}$ths of its length.

See Edinburgh Ency. Article, Centre of Percussion.

CENTRE OF GYRATION.

The centre of Oscillation already described, is the point into which all the matter of a body is collected, when the body is put in motion by its own gravity; and the centre of Gyration is the point into which all the matter of a body is collected, when it is put in motion by any extraneous force.

If a straight bar, equally thick, was struck at the centre of of gyration, the stroke would communicate the same angular velocity to the bar, as if the whole bar was collected in that point.

The force of any particle revolving round a centre, is, as that particle multiplied by the square of its velocity, or of its distance from the centre of motion; consequently, the force required to destroy that motion must be equal to it.

For example; suppose a bar of a uniform density 12 feet long, and each foot weighing 7 libs, and revolving from a centre 3 feet from the one end; at what distance will the centre of gyration be from the centre of motion?

$a = 9$ feet long end.
$b = 3$ do. short end.
$c = 7$ libs each foot.

$9 \times 7 = 63$, weight of long end.
$3 \times 7 = 21$, weight of short end.
$63 \times 9^2 = 5103$, force of long end.
$21 \times 3^2 = 189$, do. of short end.

Centre of gyration $\sqrt{\dfrac{5103+189}{3 \times \overline{63+21}}} = 4.5$ feet from centre of motion.*

For another example—Let the same bar have a weight of 50 libs at each end, then at what distance will the centre of gyration be from the centre of motion?

a = 9 feet, long end.
b = 3 do. short end.
c = 7 libs each foot.
d = 50 libs weight at long end.
e = 50 do. at short end.
z = distance of centre of gyration from centre of motion.

$63 \times 9^2 = 5103$, force of long end.
$21 \times 3^2 = 189$, do. of short end.
$50 \times 9^2 = 4050$, do. of weight on long end.
$50 \times 3^2 = 450$, do. of weight on short end.

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*$\dfrac{ac.a^2}{3} + \dfrac{bc.b^2}{3} = \overline{ac+bc} \times z^2$ or $z = \sqrt{\dfrac{ac.a^2+bc.b^2}{3\,ac+bc}}$ z being the centre gyration.

$$\sqrt{\frac{3\times\overline{4050+450}+5103+189}{3\times\overline{50+50+63+21}}}=\sqrt{\frac{18792}{552}}=5.84$$

distance between the centres.*

The centre of gyration, with respect to practical utility, is the same as the two foregoing centres. The following Rule is the easiest mode of ascertaining the centre of gyration.

"If the distance of the centre of oscillation from the centre of the system, or point of suspension, be multiplied by the distance of the centre of gravity from the same point, the square root of the product will be the distance of the centre of gyration, *i. e.* let the centre of gravity be 4, and the centre of oscillation 9, then $4\times9=36$, and the square root of that is 6; therefore 6 is the distance that the centre of gyration is from the point of suspension."

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* $da^2+\frac{ca^3}{3}+eb^2+\frac{cb^3}{3}$ = the force of the whole revolving round the centre of motion, and which must be equal to $\overline{ac+d+bc+e}\times Z^2$, therefore

$$Z^2\times\overline{ac+d+bc+e}=da^2+eb^2+\frac{ca^3+cb^3}{3}\text{, or}$$

$$Z^2=\frac{\overline{3da^2+eb^2+ca^3+cb^3}}{3ac+d+bc+e}\text{, and}$$

$$Z=\sqrt{\frac{\overline{3da^2+eb^2+ca^3+cb^3}}{3ac+d+bc+e}}$$

The following note is given by Dr. Hutton, in the 3d vol. of his Math. art. max. of Machines, p. 244.

The distance of R, the centre of gyration, from C the centre or axis of motion, in some of the most useful cases, is as below.

In a circular wheel of uniform thickness	$CR = \text{rad.} \sqrt{\frac{1}{2}}$.
In the periphery of a circle revolving about the diameter .	$CR = \text{rad.} \sqrt{\frac{1}{2}}$.
In the plane of a circle ditto .	$CR = \frac{1}{2}$ rad.
In the surface of a sphere ditto .	$CR = \text{rad.} \sqrt{\frac{2}{3}}$.
In a solid sphere . . ditto .	$CR = \text{rad.} \sqrt{\frac{2}{5}}$.
In a plane ring formed of circles whose radii are R, r, revolving about centre.	$CR = \sqrt{\frac{R^4}{2R^2 - 2r^2}}$.
In a cone revolving about its vertex	$CR = \frac{1}{2}\sqrt{\frac{12}{5}a^2 + \frac{3}{5}r^2}$.
In a cone . . . . . . its axis .	$CR = r\sqrt{\frac{3}{15}}$.
In a straight lever whose arms are R and r	$CR = \sqrt{\frac{R^3 + r^3}{3(R+r)}}$

SUMMARY.

If P be any particle of a body B, and d its distance from the axis of motion S, also G O R the centres of Gravity, Oscillation, and Gyration. Then the centres of

$$\text{Gravity will be} = \frac{Pd}{B} = G.$$

$$\text{Percussion do.} = \frac{Pd^2}{SGB} = O.$$

$$\text{Gyration do.} = \sqrt{\frac{Pd^2}{B}} = R.$$

For ample Explanations and Examples of the foregoing Centres, *See Hutton's Mathematics, Banks on Mills, &c.*

CENTRAL FORCES.

1. The quantity of matter in a body is as its magnitude and density; that is if a body measures 7 cubic feet, and a cubic foot weighs 10 libs, the quantity of matter in that body will be $= 7 \times 10 = 70$ libs.

2. All bodies naturally endeavour to continue in their present state, whether of rest or motion.

3. When a body at rest is struck by a force so as to produce motion, that motion is in proportion to the force, and in the direction of the right line in which it acts.

4. Action and Reaction between any two bodies, are equal and contrary; that is by Action and Reaction equal changes of motion are produced in bodies acting on each other; and these changes are directed towards opposite or contrary parts.

5. The Momenta, or quantities of motion in moving bodies, are as their masses and velocities; for a body of 20 libs, moving at a velocity of 10 feet, will have a momentum of 200; but a body of 6 libs, moving at the same velocity, will have only 60 for its momentum.

6. A body moving round a central point inclines to fly off in a straight line, from the first impulse of

motion: the force which causes it to leave that line, or move in a circle round the point, is called the Centripetal; and the resistance which it affords, the Centrifugal force; or, in other words, when a body revolves round its centre of motion, the centrifugal force is that power or tendency which the body has to burst or fly asunder; and the centripetal force, is that power which keeps the body from bursting or flying asunder.

It is evident from the last remark, that the greater the velocity, the greater will be the centrifugal force; and from the 5th remark, the greater the mass, the greater the momentum; therefore, as is the weight and velocity of the revolving body, so is the centrifugal force.

Suppose two fly-wheels of the same weight, one of them 12 feet diameter, and revolving in 8 seconds; what must be the diameter of the other, when it revolves in 3 seconds?

The diameter and velocity of the first, must be equal to the diameter and velocity of the second; therefore as $8^2 : 12 :: 3^2$ to the diameter $= \frac{12 \times 3^2}{8^2}$

$= \frac{108}{64} = 1.6875$ foot, the diameter of the second fly-wheel at the circle of percussion.

Again, suppose two fly-wheels of the same diameter, the one revolving in 3 seconds, and the other in 8 seconds; what will be the difference of their weights?

As $3^2 : 8^2$ so is the weight of the one, to the weight of the other.

$\frac{8^2}{3^2}=\frac{64}{9}=7\frac{1}{9}$, their weights will be to each other as $7\frac{1}{9}$ is to 1; and by knowing the weight of the second, and dividing it by $7\frac{1}{9}$, will give the weight of the first.

In the two preceding Examples, weight and velocity are taken separately.—The following Examples give the centrifugal force, when both weight and velocity are used.

Required the centrifugal force of a fly-wheel, diameter 16 feet, velocity 50 revolutions per minute, and weight $3\frac{1}{2}$ tons?

3.1416=circumference of a circle the diameter 1.

16 feet=space a body falls through in 1 second of time.

$.83\dot{3}$=time of one revolution.

$\frac{16 \times 3.1416^2}{16 \times .833^2}=\frac{157.9136}{11.1122}=14.21$ times the weight in tons, the wt. being $3\frac{1}{2}$ tons; therefore $3.5 \times 14.21 = 49.73$ tons, the centrifugal force.

The stones on which they grind table knives at Sheffield, are about 44 inches diameter, and weigh about half a ton; the velocity of the circumference is at the rate of 1250 yards in a minute; equal to 326 revolutions; required the centrifugal force?

$22^2 \times 2=968$, the square root of which is 31.1 inches, or 2.59 feet, the diameter of the circle of gyration.

As 326 : 60 : : 1 : .184 seconds, the time of one revolution.

$\frac{2.59 \times 3.1416^2}{16 \times .184^2} = \frac{25.5622}{.54169} = 47.18$ times the weight of the stone: the stone is .5 ton, therefore $47.18 \times .5 = 23\frac{1}{2}$ tons centrifugal force.

MOTION, RESISTANCE, AND EFFECT OF MACHINES.

Various as the modifications of Machines are, and innumerable their different applications; still there are only three distinct objects to which their utility tends.

The first is, in furnishing the means of giving to the moving force the most commodious direction; and when it can be done, of causing its action to be applied immediately to the body to be moved. These can rarely be united: but the former can be accomplished in most cases.

The second, in accommodating the velocity of the work to be performed, to the velocity with which alone a natural power can act.

The third, and most essential advantage of machines, is in augmenting, or rather in modifying the energy of the moving power, in such a manner, that

it may produce effects of which it would have been otherwise incapable. For instance, a man might with exertion lift 4 cwt.; but let him apply a lever, and he will lift many times that weight.

The motions produced by machines are of three kinds, viz. Accelerated, Uniform, and Alternate, *i. e.* accelerated and retarded. The first of these always takes place when the moving power is immediately applied; the second, after the machine has been in motion for a short time; the third, in intermitting machines, such as pendulum clocks, &c.; but though a second's pendulum is accelerated the first half second and retarded the next; still it produces a constant number of vibrations in a given time, and therefore may be considered as a machine of uniform motion.

The grand object in all practical cases, is, to procure a uniform motion, because it produces the greatest effect. All irregularities of motion indicate that there is some point resisting the motion, and to overcome which a part of the propelling power is wasted, and the greatest varying velocity is only equal to that velocity by which the machine would move when its motion is uniform. If the machine moves with an accelerating velocity, it is certain that the power is greater than what balances the opposing resistance, and therefore cannot produce the greatest effect; because the whole resistance is not applied. In both these cases the machine has neither the power nor the effect which it would have if moving uniformly.

When irregularity of motion takes place, particularly in a large heavy machine, it suffers a continual straining and jolting, which must very soon destroy it. It is therefore of the greatest consequence, that, from all machines, every cause tending to produce irregularity of motion should be taken away.

The fundamental Rules already given, and the annexed statement respecting the motion of machines, render any elaborate calculation on the maximum and minimum effects of motion unnecessary.

STEAM ENGINE.

The Rules of Practical Mechanics, with various Examples, have been stated, embracing the Mechanical Powers, which exhibit the Analysis of Machinery—The Rules for finding the weight and strength of the different parts—The centres of Gravity, Percussion, and Gyration, which determine, according to the nature of the machine, that point in which all its force is collected—And an explanation of the nature of Motions, and the effect of Machines; showing, that when a Machine is producing the greatest effect, or working at a maximum, that the motion is uniform; the power and resistance being in a proper proportion to each other.

What remains now to be explained, are, the Rules connected with the Steam Engine, Water Wheel, Common and Force Pumps, which are, viz.

STEAM ENGINE.

Fuel.—To produce equal heats, $\frac{3}{4}$ cwt. of Newcastle Coal is equal to 1 cwt. of Glasgow Coal, and to $2\frac{1}{4}$ cwt. of Wood, or three times the weight.—Also, it takes double the weight of Culm to that of Coal.

Upon the premises of Messrs. Claud Girdwood & Co. of Glasgow, there are two Steam Engines, one of 32 horse power and one of 12 horse power. The daily consumpt of Culm for the first, upon an average, is 5 wagons, 24 cwt. each; for the second 1½ wagon. Taking an average from these two, gives 34 libs of Culm per hour for each horse power.

The Cornish statement of the effect of a bushel of Coal produced by a Boulton and Watt Engine, is = 28,000,000 libs lifted 1 foot. Using high pressure steam to a B. and W. Engine (say steam at 40 libs.) is =35,000,000 libs.

Data.—A 10 horse B. and W. Engine consumes 1 bushel of Coal per hour, which is equal to $44000 \times 10 \times 60 = 26,400,000$ libs. The medium consumption of Newcastle Coal is for a 10 horse Engine, 12 bushels in 10 hours, or 1.2 bushel per hour, (say from 10 to 12 libs per hour) for one horse.

Scotch Coal, 17 libs per hour for 1 horse.
Newcastle 12 do. do. 1 do.

Note. The London bushel is 87 libs.

I trust I shall not be thought impertinent, and, I hope not partial, in recommending one of the latest, and I may safely say, the best improvement as yet known, for the economical using of fuel; it is, Mr. Brunton's Fire Regulator. This machine feeds the fire in the most regular manner, and nicely

proportions the quantity of Coal thrown upon the grate to the quantity of steam required.

Almost the whole of the Public Works, using Steam Engines in London, have this Fire Regulator attached to their Boilers. And the accounts kept by their Engineers, of the quantity of Coal consumed, exhibit a saving of from 15 to 25 per cent. produced by it. But the regular manner of feeding the fire, and consequently, the saving of fuel, are not the only advantages derived from it. There is no regular fireman needed, the Hopper only requires to be filled with Coal in the morning, and no other attendance is necessary; also the supplementary Boiler, which is attached to the large Boiler, gives an additional quantity of Steam, say from 2 to 6 horses, in proportion to the size of the Engine, and preserves the large Boiler from the injurious effects of the fire.

These advantages, derived by this Fire Regulator over the usual mode of feeding the fire by hand, make it one of the most useful inventions of the present day, and, in fact, a Steam Engine is not complete without it.

Boilers—are of various forms, but the most general is proportioned as follows, viz. width 1, depth 1.1, length 2.5; their capacity being, for the most part, two horse more than the power of the Engine for which they are intended.

Boulton and Watt allow 25 cubic feet of space for each horse power, some of the other Engineers allow 5 feet of surface of water.

STEAM—arising from water at the boiling point, is equal to the pressure of the atmosphere, which is in round numbers, 15 libs on the square inch; but to allow for a constant and uniform supply of Steam to the Engine, the safety valve of the Boiler is loaded with three libs on each square inch.

The following Table exhibits the expansive force of Steam, expressing the degrees of heat at each lib of pressure on the safety valve.

Degrees of Heat.	Libs of Pressure.	Degrees of Heat.	Libs of Pressure.	Degrees of Heat.	Libs of Pressure.
212°	0	268°	24	298°	48
216	1	270	25	299	49
219	2	271	26	300	50
222	3	273	27	301	51
225	4	274	28	302	52
229	5	275	29	303	53
232	6	277	30	304	54
234	7	278	31	305	55
236	8	279	32	306	56
239	9	281	33	307	57
241	10	282	34	308	58
244	11	283	35	309	59
246	12	285	36	310	60
248	13	286	37	311	61
250	14	287	38	312	62
252	15	288	39	313	63
254	16	289	40	313½	64
256	17	290	41	314	65
258	18	291	42	315	66
260	19	293	43	316	67
261	20	294	44	317	68
263	21	295	45	318	69
265	22	296	46	319	70
267	23	297	47	320	71

By the following Rule the quantity of steam required to raise a given quantity of water to any given temperature is found.

Rule. Multiply the water to be warmed by the difference of temperature between the cold water and that to which it is to be raised, for a dividend, then to the temperature of the steam add 900 degrees, and from that sum take the required temperature of the water: this last remainder being made a divisor to the above dividend, the quotient will be the quantity of steam in the same terms as the water.

EXAMPLE.

What quantity of steam at 212° will raise 100 gallons of water at 60° up to 212°?

$\frac{212^\circ - 60^\circ \times 100}{212^\circ + 900^\circ - 212^\circ} = 17$ gallons of water formed into steam.

Now, steam at the temperature of 212° is 1800 times its bulk in water; or 1 cubic foot of steam, when its elasticity is equal to 30 inches of mercury, contains 1 cubic inch of water.—Therefore 17 gallons of water converted into steam, occupies a space of 4090¾ cubic feet, having a pressure of 15 libs on the square inch.

In boiling by steam, using a jacket instead of blowing the steam into the water, I believe, about 10.5 square feet of surface are allowed for each horse capacity of boiler—*i. e.* a 14 horse boiler will boil water in a pan set in a jacket, exposing a surface of $10.5 \times 14 = 147$ square feet.

HORSE POWER.—Boulton and Watt suppose a horse able to raise 32,000 libs avoirdupois 1 foot high in a minute.

Desaguliers makes it 27,500 libs.

Smeaton do. 22,916 do.

It is common in calculating the power of Engines, to suppose a horse to draw 200 libs at the rate of 2½ miles in an hour, or 220 feet per minute, with a continuance, drawing the weight over a pulley—now, 200 × 220 = 44000, *i. e.* 44000 libs at 1 foot per minute, or 1 lib at 44000 feet per minute. In the following table 32,000 is used.*

One horse power is equal to raise —— gallons or —— libs —— feet high per minute.

Feet High Per Minute.	Ale Gallons.	Libs Avoirdupois.	Feet High Per Minute.	Ale Gallons.	Libs Avoirdupois.
1	3123	32000	20	156	1600
2	1561½	16000	25	125	1280
3	1041	10666	30	104	1066
4	780	8000	35	89	914
5	624	6400	40	78	800
6	520	5333	45	69	711
7	446	4571	50	62	640
8	390	4000	55	56	582
9	347	3555	60	52	533
10	312	3200	65	48	492
11	284	2909	70	44	457
12	260	2666	75	41	426
13	240	2461	80	39	400
14	223	2286	85	37	376
15	208	2133	90	34	355
16	195	2000	95	32	337
17	183	1882	100	31	320
18	173	1777	110	28	291
19	164	1684	120	26	267

*American Engineers usually assume a horse power as equivalent to 33,000 libs, raised 1 foot high per minute.

LENGTH OF STROKE.—The stroke of an Engine is equal to one revolution of the crank shaft, therefore double the length of the cylinder. When stating the length of stroke, the length of cylinder is only given, that is, an Engine with a 3 feet stroke, has its cylinder 3 feet long, besides an allowance for the piston.

The following Table shows the length of stroke, (or length of cylinder,) and the number of feet the piston travels in a minute, according to the number of strokes the Engine makes when working at maximum.

When calculating the power of Engines, the feet per minute are generally taken at 220.

Length of Stroke.	Number of Strokes.	Feet per Minute.
Feet 2	43	172
— 3	32	192
— 4	25	200
— 5	21	210
— 6	19	228
— 7	17	238
— 8	15	240
— 9	14	250

CYLINDER.—When an Engine in good order is performing its regular work, the effective pressure may be taken at 8 libs on eaeh square inch of the surface of the piston.

In the former edition the maximum effective pressure was stated at 10 libs, but few Engines are seldom or ever required to produce this work.

To calculate the power of an Engine.

RULE 1. Multiply the area of cylinder by the effective pressure = say 8 libs, the product is the weight the Engine can raise.—Multiply this weight by the number of feet the piston travels in one minute, which will give the momentum, or weight, the Engine can lift 1 foot high per minute; divide this momentum by a horse power, as previously stated, and the quotient will be the number of horse power the Engine is equal to.

RULE 2. 25 inches of the area of cylinder is equal to one horse power, the velocity of the Engine being constantly 220 feet per minute.

EXAMPLE. I.

What is the power of an Engine, the cylinder being 42 inches diameter, and stroke 5 feet?

$$\frac{42^2 \times .7854 \times 10 \times 210}{44000} = 66.12 \text{ horse power.}$$

EXAMPLE II.

What size of cylinder will a 60 horse power Engine require, when the stroke is 6 feet?

$$\frac{44000 \times 60}{228 \times 10} = 1158 \text{ inch. area of cylinder.}$$

Note. To find the power to lift a weight at any velocity, multiply the weight in libs by the velocity in feet, and divide by the horse power; the quotient will be the number of horse power required.

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TABLE.

When the effective pressure on each inch of piston is	The area equal to one horse power will be
53 libs.	3.7 inches.
48 —	4.17 —
43 —	4.65 —
38 —	5.26 —
33 —	6.06 —
28 —	7.14 —
23 —	8.7 —
18 —	11.11 —
13 —	15.46 —
8 —	25. —

NOZLES.—The diameter of the valves of Nozles ought to be fully one-fifth of the diameter of cylinder.

AIR-PUMP.—The solid contents of the Air-Pump is equal to the fourth of the solid contents of cylinder, or when the Air-Pump is half the length of the stroke of the Engine, its area is equal to half the area of cylinder.

CONDENSER—is generally equal in capacity to the Air-Pump; but when convenient, it ought to be more; for when large, there is a greater space of vacuum, and the steam is sooner condensed.

COLD WATER PUMP.—The capacity of the Cold Water Pump depends on the temperature of the water. Many Engines return their water, which cannot be so cold as water newly drawn from a river,

well, &c.; but when water is at the common temperature, each horse power requires nearly 7½ gallons per minute.* Taking this quantity as a standard, the size of the pump is easily found by the following Rule, viz.—Multiply the number of horse power by 7½ gallons, and divide by the number of strokes per minute; this will give the quantity of water to be raised each stroke of pump. Multiply this quantity by 231, (the number of cubic inches in a gallon,) and divide by the length of effective stroke of pump, the quotient will be the area.

EXAMPLE.

What diameter of pump is requisite for a 20 horse power Steam Engine having a 3 feet stroke, the effective stroke of pump to be 15 inches?

$20 \times 7\frac{1}{2} = \frac{150}{32} = 4.6875$ gallons the pump lifts each stroke.

$\frac{4.6875 \times 231}{15} = 72.1875$ inches area of pump.

Hot Water Pump.—The quantity of water raised at each stroke ought to be equal in bulk to the 900th part of the capacity of the cylinder.

* An Engine will work with a less supply of water, say 5 gallons per minute; but when water can be had without a considerable expense of power, 7½ gallons is preferable; because an abundance of water keeps the condenser, &c. cool, and thereby produces a better vacuum.

Proportions.—The length of stroke being 1, the length of beam to centre will be 2, the length of crank .5, and the length of connecting rod 3.

The following Table shows the force which the connecting rod has to turn round the crank at different parts of the motion.

Col. A. Decimal proportions of descent of the Piston, the whole descent being 1.

Col. B. Angle between the connecting Rod and Crank.

Col. C. Effective length of the Lever upon which the connecting Rod acts, he whole Crank being 1.

Col. D. Decimal proportions of half a revolution of the Fly-Wheel.

Col. C. Also shows the force which is communicated to the Fly-Wheel, expressed in decimals, the force of the Piston being 1.

A	B	C	D
.0	180°	.0	.0
.05	151½	.46	.128
.10	141	.62	.158
.15	131½	.74	.228
.2	123½	.830	.271
.25	117¼	.892	.308
.3	110¾	.94	.342
.35	104	.976	.377
.4	97½	.986	.41
.45	91¾	1.	.441
.5	85½	1.	.473
.55	80	.986	.507
.6	75	.956	.538
.65	69	.92	.572
.7	62½	.88	.607
.75	57½	.824	.642
.8	49	.746	.68
.85	42	.66	.723
.9	34	.546	.776
.95	23½	.390	.84
1.0	0	.000	1.0

Fly-Wheel—Is used to regulate the motion of the Engine, and to bring the crank past its centres. The Rule for finding its weight, is,—Multiply the number of horses' power of the Engine by 2000, and divide by the square of the velocity of the circumference of the wheel per second, the quotient will be the weight in cwts.

EXAMPLE.

Required the weight of a fly-wheel proper for an Engine of 20 horse-power, 18 feet diameter, and making 22 revolutions per minute?

18 feet diameter $= 56$ feet circumference, $\times$ 22 revolutions per minute $= 1232$ feet, motion per minute $\div 60 = 20\frac{1}{2}$ feet motion per second; then $20\frac{1}{2}^2 = 420\frac{1}{4}$ the divisor.

20 horse power $\times 2000 = 40000$ dividend.

$$\frac{40000}{420\frac{1}{4}} = 90.4 \text{ cwt. weight of wheel.}$$

PARALLEL MOTION.—The radius and parallel bars are of the same dimensions; their length being generally 1-4th of the length of the beam between the two glands, or one-half of the distance between the fulcrum and gland. Both pairs of straps are the same length between the centres, and which is generally three inches less than the half of the length of stroke. *See Plate 3d.*

GOVERNOR, or DOUBLE PENDULUM—If the revolutions be the same, whatever be the length of the arms, the balls will revolve in the same plane, and the distance of that plane from the point of suspension, is equal to the length of a pendulum, the vibrations of which will be double the revolutions of the balls. For example; suppose the distance between the point of suspension and plane of revolution be 36

inches, the vibrations that a pendulum of 36 inches will make per minute, is$=\frac{375}{\sqrt{36}}=$ 62 vibrations, and $\frac{62}{2}=31$ revolutions per minute the balls ought to make.

WATER WHEEL.

THIS subject belongs to Hydrodynamics, also the common and force Pumps; and since they are the last of this Treatise, they may be classed under that name, to distinguish them from the preceding subjects in Statics and Dynamics.

WATER. (*Hydrostatics.*)

Hydrostatics is the science which treats of the pressure, or weight, and equilibrium of water, and other fluids, especially those that are non-elastic.

Note 1. The pressure of water at any depth, is as its depth; for the pressure is as the weight, and the weight is as the height.

Note 2. The pressure of water on a surface any how immersed in it, either perpendicular, horizon-

tal or oblique, is equal to the weight of a column of water, the base being equal to the surface pressed, and the altitude equal to the depth of the centre of gravity, of the *surface pressed*, below the top or surface of the fluid.

PROBLEM I.

In a vessel filled with water, the sides of which are upright and parallel to each other, having the top of the same dimensions as the bottom, the pressure exerted against the bottom, will be equal to the area of the bottom multiplied by the depth of water.

EXAMPLE.

A vessel 3 feet square and 7 feet deep, is filled with water; what pressure does the bottom support?

$$\frac{3^2 \times 7 \times 1000}{16} = 3937\tfrac{1}{2} \text{ libs Avoirdupois.}$$

PROBLEM II.

A side of any vessel sustains a pressure equal to the area of the side multiplied by half the depth, therefore the sides and bottom of a cubical vessel sustain a pressure equal to three times the weight of water in a vessel.

EXAMPLE I.

The gate of a sluice is 12 feet deep and 20 feet broad; what is the pressure of water against it?

$$\frac{20 \times 12 \times 6 \times 1000}{16} = 90000 = 40\tfrac{1}{5} \text{ tons nearly.}$$

From Note 2d.—The pressure exerted upon the side of a vessel, of whatever shape it may be, is as the area of the side and centre of gravity below the surface of water.

EXAMPLE II.

What pressure will a board sustain, placed diagonally through a vessel, the side of which is 9 feet deep, and bottom 12 feet by 9 feet?

$\sqrt{12^2+9^2}=15$ feet, the length of diagonal board.

$$\frac{15\times9\times4\frac{1}{2}\times1000}{16}=37969 \text{ libs nearly.}$$

Though the diagonal board bisects the vessel, yet it sustains more than half of the pressure in the bottom, for the area of bottom is 12×9, and the half of the pressure is $\frac{1}{2}$ $60750=30375$.

The bottom of a conical or pyramidical vessel sustains a pressure equal to the area of the bottom and depth of water, consequently, the excess of pressure is three times the weight of water in the vessel.

Water. (*Hydraulics.*)

Hydraulics is that science which treats of fluids considered as in motion, it therefore embraces the phenomena exhibited by water issuing from orifices in reservoirs, projected obliquely, or perpendicularly, in *Jet-d'eaux*, moving in pipes, canals, and rivers, oscillating in waves, or opposing a resistance to the progress of solid bodies.

It would be needless here to go into the minutiæ of hydraulics, particularly when the theory and practice do not agree. It is only the general laws, deduced from experiment, that can be safely employed in the various operations of hydraulic architecture.

Mr. Banks, in his treatise on Mills, after enumerating a number of experiments on the velocity of flowing water, by several philosophers, as well as his own, takes from thence the following simple rule, which is as near the truth as any that have been stated by other experimentalists.

RULE. Measure the depth (of the vessel, &c.) in feet, extract the square root of that depth, and multiply it by 5.4, which gives the velocity in feet per second; this multiplied by the area of the orifice in feet, gives the number of cubic feet which flows out in one second.

EXAMPLE.

Let a sluice be 10 feet below the surface of the water, its length 4 feet, and open 7 inches; required the quantity of water expended in one second?

$\sqrt{10} = 3.162 \times 5.4 = 17.0748$ feet velocity.

$\frac{4 \times 7}{12} = 2\frac{1}{3}$ feet $\times\ 17.0748 = 39.84$ cubic feet of water per second.

If the area of the orifice is great compared with the head, take the medium depth, and two thirds of the velocity from that depth, for the velocity.

EXAMPLE.

Given the perpendicular depth of the orifice 2 feet, its horizontal length 4 feet, and its top 1 foot below the surface of water. To find the quantity discharged in one second:

The medium depth is $= 1.5 \times 5.4 = 8.10 - \frac{1}{3} = 5.40 \times 8 = 43.20$ cubic feet.*

The quantity of water discharged through slits, or notches, cut in the side of a vessel or dam, and open at the top, will be found by multiplying the velocity at the bottom by the depth, and taking $\frac{2}{3}$ of the product for the area; which again multiplied by the breadth of the slit, or notch, gives the quantity of cubic feet discharged in a given time.

EXAMPLE.

Let the depth be 5 inches, and the breadth 6 inches; required the quantity run out in 46 seconds?

The depth is .4166 of a foot.

The breadth is .5 of a foot.

$\sqrt{.4166} = .6455 \times 5.4 \times \frac{2}{3} = 2.3238 \times .4166 = .96825 \times .5 = .48412$ feet per second.

Then $.48412 \times 46. = 22.269$ cubic feet in 46 seconds.

There are two kinds of water wheels, Undershot and Overshot. Undershot when the water strikes the wheel at, or below the centre. Overshot, when the water falls upon the wheel above the centre.

* The square root of the depth is not taken in this example, but when the depth is considerable, it ought to be taken.

The effect produced by an *undershot* wheel, is from the impetus of the water. The effect produced by an *overshot* wheel, is from the gravity or weight of the water.

Of an undershot wheel, the power is to the effect as 3 : 1.—Of an overshot wheel, the power is to the effect as 3 : 2—which is double the effect of an undershot wheel.

The following is an Abridgement of SMEATON *on* WATER WHEELS.

UNDERSHOT.

Velocity of water in 1″ .	=V	
Weight of 1 cub. in. of water	=W	V.A= Q in one second.
Area of sluice . . .	=A	$\overline{\text{QW}}$.V=P; Power to produce mechanical effect.
Quantity of water . .	=Q	
Power of the water to produce mechanical effect	= P	

POWER AND EFFECT AT MAXIMUM.

Velocity of Wheel in 1″ .	=v		
Effective velocity of water	=E	V—v =E	P : *e* : : 10 : 3.62
Effect produced by the wheel	=*e*	*wv* = *e*	or 3 : 1
Weight raised	=*w*		V : v : : 10 : 3.5,
Velocity of weight raised .	=*v*		or 5 : 2

OVERSHOT.

Descent of water including head and diameter of wheel*	=D	D.W=P.
The weight of water expended in one second	=W	

POWER AND EFFECT AT MAXIMUM.

Power of the water is =D.W=P	P : *e* : : 10 : 6.6, or 3 : 2 nearly.
Effect of the wheel is =*wv*=*e*	Double that of an Undershot.

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* By Head is understood the distance between the orifice and the part of the wheel on which the water falls. The fall is the perpendicular height from the bottom of the wheel to the orifice.

The velocity at a maximum is = 3 feet in one second.

Since the effect of the overshot is double that of the undershot, it follows that the higher the wheel is in proportion to the whole descent, the greater will be the effect.

The maximum load for an overshot wheel is that which reduces the circumference of the wheel to its proper velocity, = 3 feet in 1 second; and this will be known, by dividing the effect it ought to produce in a given time, by the space intended to be described by the circumference of the wheel in the same time; the quotient will be the resistance overcome at the circumference of the wheel, and is equal to the load required, the friction and resistance of the machinery included.

The following is an Extract from Banks on Mills, page 152.

"The effect produced by a given stream in falling through a given space, if compared with a weight, will be directly as that space; but if we measure it by the velocity communicated to the wheel, it will be as the square root of the space descended through, agreeably to the laws of falling bodies.

"*Experiment* 1. A given stream is applied to a wheel at the centre; the revolutions per minute are 38.5.

"*Ex.* 2. The same stream applied at the top, turns the same wheel 57 times in a minute.

"If in the first experiment the fall is called 1, in the second it will be 2: then $\sqrt{1} : \sqrt{2} :: 38.5 : 54.4$, which are in the same ratio as the square roots of the spaces fallen through, and near the observed velocity.

"In the following experiments a fly is connected with the water wheel.

"*Ex.* 3. The water is applied at the centre, the wheel revolves 13.03 times in one minute.

"*Ex.* 4. The water is applied at the vertex of the wheel, and it revolves 18.2 times per minute.

"As $13.03 : 18.2 :: \sqrt{1} : \sqrt{2}$ nearly.

"From the above we infer, that the circumferences of wheels of different sizes may move with velocities which are as the square roots of their diameters without disadvantage, compared one with another, the water in all being applied at the top of the wheel, for the velocity of falling water at the bottom or end of the fall is as the time, or as the square root of the space fallen through; for example, let the fall be 4 feet, then, As $\sqrt{16} : 1'' :: \sqrt{4} : \frac{1}{2}''$, the time of falling through 4 feet:—Again, let the fall be 9 feet, then, $\sqrt{16} : 1'' :: \sqrt{9} : \frac{3}{4}''$, and so for any other space, as in the following Table, where it appears that water will fall through one foot in a quarter of a second, through 4 feet in half a second, through 9 feet in 3 quarters of a second, and through 16 feet in one second. And if a wheel 4 feet in diameter moved as fast as the water, it could not revolve in less than 1.5 second, neither could a wheel of 16 feet diameter revolve in less

than three seconds; but though it is impossible for a wheel to move as fast as the stream which turns it; yet, if their velocities bear the same ratio to the time of the fall through their diameters, the wheel 16 feet in diameter may move twice as fast as the wheel 4 feet diameter."

TABLE.

Height of the fall in Feet.	Time of falling in Seconds.	Height of the fall in Feet.	Time of falling in Seconds.
1	.25	14	.935
2	.352	16	1.
3	.432	20	1.117
4	.5	24	1.22
5	.557	25	1.25
6	.612	30	1.37
7	.666	36	1.5
8	.706	40	1.58
9	.75	45	1.67
10	.79	50	1.76
12	.864		

Power and Effect.—The power water has to produce mechanical effect, is as the quantity and fall of perpendicular height.—The mechanical effect of a wheel is as the quantity of water in the buckets and the velocity.

The power is to the effect as 3 : 2, that is, suppose the power to be 9000, the effect will be $=\frac{9000. \times 2}{3}=\frac{18000}{3}=6000.$

Height of the Wheel.—The higher the wheel is in proportion to the fall, the greater will be the effect, because it depends less upon the impulse,

and more upon the gravity of the water; however, the head should be such, that the water will have a greater velocity than the circumference of the wheel; and the velocity that the circumference of the wheel ought to have being known, the head required to give the water its proper velocity, can easily be known from the rules of Hydrostatics.

VELOCITY OF THE WHEEL.—Banks, in the foregoing quotation, says, "That the circumferences of overshot wheels of different sizes may move with velocities as the square roots of their diameters, without disadvantage." Smeaton says, "Experience confirms that the velocity of 3 feet per second is applicable to the highest overshot wheels, as well as the lowest; though high wheels may deviate further from this rule, before they will lose their power, by a given aliquot part of the whole, than low ones can be admitted to do; for a 24 feet wheel may move at the rate of 6 feet per second, without losing any considerable part of its power."

It is evident that the velocities of wheels, will be in proportion to the quantity of water and the resistance to be overcome:—if the water flows slowly upon the wheel, more time is required to fill the buckets than if the water flowed rapidly; and whether Smeaton or Banks is taken as a data, the mill-wright can easily calculate the size of his wheel, when the velocity and quantity of water in a given time is known.

EXAMPLE I.

What power is a stream of water equal to, of the following dimensions, viz. 12 inches deep, 22 inches broad; velocity, 70 feet in $11\frac{3}{4}$ seconds, and fall, 60 feet?—Also, what size of a wheel could be applied to this fall?

$\frac{12 \times 22}{144} = 1.83$ square feet:—area of stream.

$11\frac{3}{4}'' : 70 :: 60'' : 357.5$ lineal feet per min.—velocity.

$357.5 \times 1.83 = 654.225$ cubic feet per minute.

$654.225 \times 62.5 = 40889.0625$ avoir. libs per minute.

$40889.0625 \times 60 = 2453343.7500$ momentum at a fall of 60 feet.

$\frac{2453343.7500}{44000} = 55.7$ horse power.

$3 : 2 :: 55.7 : 37.13$ effective power.

The diameter of a wheel applicable to this fall, will be 58 feet, allowing one foot below for the water to escape, and one foot above for its free admission.

$58 \times 3.1416 = 182.2128$ circumference of wheel.

$60 \times 6 = 360$ feet per minute, = velocity of wheel.

$\frac{654.225}{360} = 1.8$ sectional area of buckets.

The bucket must only be half full, therefore $1.8 \times 2 = 3.6$ will be the area.

To give sufficient room for the water to fill the buckets, the wheel requires to be 4 feet broad, now, $\frac{3.6}{4} = .9$, say 1 foot depth of shrouding.

$\frac{360}{182.2128} = 1.9$ revolutions per minute the wheel will make.

	Power of water . .	= 55.7 H. P.	Ans.
	Effective power of do.	= 37.13 H. P.	
Dimensions of Wheel.	Diameter . .	= 58 feet.	
	Breadth . .	= 4 feet.	
	Depth of shrouding	= 1 foot.	

EXAMPLE II.

What is the power of a water wheel, 16 feet diameter, 12 feet wide, and shrouding 15 inches deep.

$16 \times 3.1416 = 50.2656$ circumference of wheel.

$12 \times 1\frac{1}{4} = 15$ square feet, sectional area of buckets.

$60 \times 4 = 240$ lineal feet per minute, = velocity.

$240 \times 15 = 3600$ cubic feet water, when buckets are full; when half full, 1800 cubic feet.

$1800 \times 62.5 = 112500$ avoir. libs of water per minute.

$112500 \times 16 = 1800000$ momentum, falling 16 feet.

$3 : 2 :: 1800000 : \frac{1200000}{44000} = 27$ horse power.

Buckets.—The number of buckets to a wheel should be as few as possible, to retain the greatest quantity of water; and their mouths only such a width as to admit the requisite quantity of water, and at the same time sufficient room to allow the air to escape.

The Communication of Power.—There are no prime movers of machinery from which power in

taken in a greater variety of forms than the water wheel, and among such a number there cannot fail to be many bad applications.

Suffice it here to mention one of the worst, and most generally adopted. For driving a cotton mill in this neighbourhood, there is a water wheel about 12 feet broad, and 20 feet diameter; there is a division in the middle of the buckets upon which the segments are bolted round the wheel, and the power is taken from the vertex: from this erroneous application, a great part of the power is lost; for the weight of water upon the wheel presses against the axle in proportion to the resistance it has to overcome, and if the axle was not a large mass of wood, with very strong iron journals, it could not stand the great strain which is upon it.

The most advantageous part of the wheel, from which the power can be taken, is that point in the circle of gyration horizontal to the centre of the axle; because, taking the power from this part, the whole weight of water in the buckets acts upon the teeth of the wheels; and the axle of the water wheel suffers no strain.

The proper connexion of machinery to water wheels is of the first importance, and mismanagement in this particular point is often the cause of the journals and axles giving way, besides a considerable loss of power.

To find the radius of the circle of gyration in a water wheel is therefore of advantage to the saving of power, and the following Example will show the rule by which it is found. *See Centre of Gyration.*

EXAMPLE.

Required the radius of the circle of gyration in a water wheel, 30 feet diameter; the weight of the arms being 12 tons, shrouding 20 tons, and water 15 tons.

30 feet diameter, radius = 15 feet.

$$S\ 20\times15^2 = 4500\times2=9000$$
$$A\ \frac{12\times15^2}{3} = 900\times2=1800$$

The opposite side of the water wheel must be taken.

$$W\ 15\times15^2=3375 \qquad =3375$$
$$2\times\overline{20+12}=64 \qquad \overline{14175}$$
$$W\ \ 15$$
$$\overline{79} \qquad \frac{14175}{79}=179,$$ the square root

of which is 13 $\frac{4}{10}$ feet, the radius of the circle of gyration.

PUMPS.

There are two kinds of Pumps, Lifting and Forcing. The Lifting, or Common Pumps, are applied to wells, &c. where the depth does not exceed 32 feet; for beyond this depth they cannot act, because the height that water is forced up into a vacuum, by the pressure of the atmosphere, is about 34 feet.

The Force Pumps are those that are used on all other occasions, and can raise water to any required

height.—Bramah's celebrated Pump is one of this description, and shows the amazing power that can be produced by such application, and which arises from the fluid and non-compressible qualities of water.

The power required to raise water any height is equal to the quantity of water discharged in a given time, and the perpendicular height.

EXAMPLE.

Required the power necessary to discharge 175 Ale gallons of water per minute, from a pipe 252 feet high?

One Ale gallon of water weighs $10\frac{1}{4}$ libs avoir. nearly.

$$175 \times 10\tfrac{1}{4} = 1799 \times 252 = \frac{453348}{44000} = 10.3 \text{ horse power.}$$

The following is a very simple Rule, and easily kept in remembrance.

Square the diameter of the pipe in inches, and the product will be the number of libs of water avoirdupois contained in every yard length of the pipe. If the last figure of the product be cut off, or considered a decimal, the remaining figures will give the number of Ale gallons in each yard of pipe; and if the product contains only one figure, it will be tenths of an Ale gallon. The number of Ale gallons multiplied by 282, gives the cubic inches in each yard of pipe; and the contents of a pipe may be found by Proportion.

EXAMPLE.

What quantity of water will be discharged from a pipe 5 inches diameter, 252 feet perpendicular height, the water flowing at the rate of 210 feet per minute?

$5^2 \times \frac{210}{3} = 175$ Ale gallons per minute.

$5^2 \times \frac{252}{3} = 2100$ libs water in pipe.

$\frac{2100 \times 210}{44000} =$ 10 horse power required to pump that quantity of water.

The following Table gives the contents of a pipe one inch diameter, in weight and measure, which serves as a standard for pipes of other diameters, their contents being found by the following Rule.

Multiply the numbers in the following Table against any height, by the square of the diameter of the pipe, and the product will be the number of cubic inches avoirdupois ounces, and Wine gallons of water, that the given pipe will contain.

EXAMPLE.

How many Wine gallons of water is contained in a pipe 6 inches diameter, and 60 feet long?

$2.4480 \times 36 = 88.1280$ Wine gallons.

In a Wine gallon there are 231 cubic inches.

TABLE.

ONE INCH DIAMETER.			
Feet High.	Quantity in Cubic Inches.	Weight in Avoir. Oz.	Gallons Wine Measure.
1	9.42	5.46	.0407
2	18.85	10.92	.0816
3	28.27	16.38	.1224
4	37.70	21.85	.1632
5	47.12	27.31	.2040
6	56.55	32.77	.2448
7	65.97	38.23	.2423
8	75.40	43.69	.3264
9	84.82	49.16	.3671
10	94.25	54.62	.4080
20	188.49	109.24	.8160
30	282.74	163.86	1.2240
40	376.99	218.47	1.6300
50	471.24	273.09	2.0400
60	565.49	327.71	2.4480
70	659.73	382.33	2.8560
80	753.98	436.95	3.2640
90	848.23	491.57	3.6700
100	942.48	546.19	4.0800
200	1884.96	1092.38	8.1600

The resistance arising from the friction of water flowing through pipes, &c. is directly as the velocity of the water, and inversely as the circumference of the pipe.

The data given is a medium, and which is 1-5th of the whole resistance: this is the standard generally adopted, being considered as most correct.

EXAMPLE I.

What is the power requisite to overcome the resistance and friction of a column of water 4 inches diameter, 100 feet high, and flowing at the velocity of 300 feet per minute?

$$\frac{546.19 \times 4^2}{16} = 546.19, \text{ say } 546.2$$

$\dfrac{546.2 \times 300}{44000} = 3.7$ $\frac{1}{5}$th of which is .7, therefore the power required to overcome the resistance occasioned by the weight and friction of the water will be $3.7 + .7 = 4.4$ H.P., say 4.5 horse power.

EXAMPLE II.

There is a cistern 20 feet square, and 10 feet deep, placed on the top of a tower 60 feet high, what power is requisite to fill this cistern in 30 minutes, and what will be the diameter of the pump, when the length of stroke is 2 feet, and making 40 per minute?

$20 \times 20 \times 10 = 4000$ cubic contents of cistern.

$\dfrac{4000}{30} = 133.3$ cubic feet of water per min.

$\dfrac{133.3 \times 1000}{16} = 8331.25$ libs avoir. per minute.

$\dfrac{8331.25 \times 60}{44000} = 11.36$ horse power, 1-5th of which is $= 2.27 + 11.11 = 13.63$ horse power required.

$\dfrac{133.3}{2 \times 40 = 80} = 1.7 \times 144 = \dfrac{244.80}{.7854} = 311.7$, now

$\sqrt{311.7} = 17.6$ inches diameter of pump required.

Founders generally prove the pipes they cast to stand a certain pressure, which is calculated by the weight of a perpendicular column of water, the area being equal to the area of the pipe, and the height equal to any given height.

To ascertain the exact pressure of water to which a pipe is subjected, a safety valve is used, generally of 1 inch diameter, and loaded with a weight equal to the pressure required: for example, a pipe requires to stand a pressure of 300 feet, what weight will be required to load the safety-valve one inch diameter?

$$\overset{\text{Feet.}}{300}\times 12=\overset{\text{Inches.}}{3600}\times .7854=\frac{2827.4400\times 1000}{1728}=\frac{\overset{\text{Ounces.}}{1636\tfrac{1}{4}}}{16}$$

$=$ 102 libs $4\tfrac{1}{4}$ oz. weight required.

Each of the weights for the safety-valves of these Hydrostatic proving-machines are generally made equal to a pressure of a column of water 50 feet high, the area being the area of the valve.

50 feet of pressure	on a	valve	1 inch	diam.	=17.06 libs
50 do.	do.	do.	$1\tfrac{1}{4}$	do.	=26.65 do.
50 do.	do.	do.	$1\tfrac{1}{2}$	do.	=38.38 do.
50 do.	do.	do.	2	do.	=68.24 do.

In pumping, there is always a deficiency owing to the escape of water through the valves; to account for this loss, there is an allowance of 3 inches for each stroke of piston rod: for example, a 3 feet stroke may be calculated at 2 feet 9 inches.

There is a town, the inhabitants of which amount to 12000, and it is proposed to supply it with water, from a river running through the low grounds 250 perpendicular feet below the best situation from the reservoir.

It is required to know the power of an engine capable of lifting a sufficient quantity of water, the daily supply being calculated at 10 Ale gallons to each individual: also what size of pump and pipes are requisite for such?

$12000 \times 10 = 120000$ gallons per day.

Engine is to work 12 hours, $\frac{120000}{12} = 10000$ gallons per hour.

$\frac{10000}{60} = 166.6$ gallons per minute.

The pump to have an effective stroke of $3\frac{3}{4}$ feet, and making 30 strokes per minute.

$\frac{166.6}{30} = 5.5533$ gallons each stroke.

$282 \times 5.6 = 1579.2$ cubic inches of water each stroke.

3 feet 9 in. $\frac{1579.2}{=45 \text{ in.}} = 35.1$ inches, area of pump.

$\frac{35.1}{.7854} = 44.7$, therefore $\sqrt{44.7} = 6.7$ diam. of pump.

The pipes will require to be at least the diameter of the pump; if they are a little more, the water will not require to flow so quickly through them, and thereby cause less friction.

The power of the Engine will be

166.6 gall. $\times 10\frac{1}{4}$ lb $\times 250$ feet $= 426925$ momentum.

$\frac{426925}{44000} = 9.7$, add 1-5th $= 11.64$ horse power.

$\frac{426925}{32000} = 13.3$, —— $= 15.96$ do. *Watt.*

$\frac{426925}{27500} = 15.5$, —— $= 18.6$ do. *Desaguliers.*

$\frac{426925}{22916} = 18.6$, —— $= 22.32$ do. *Smeaton.*

THE

RULES OF PROPORTION,

AND FOR THE EXTRACTION

OF THE

SQUARE AND CUBE ROOTS,

GEOMETRICALLY.

IN TWELVE PROBLEMS.

PROBLEM I.

To divide a line A B *into such proportion, as the line* C *is to the line* D.—Plate IV. Fig. 1.

Upon either end of the line A B make an angle by drawing A W; make A F equal to the line C, and F W equal to the line D, join W B, and parallel to that, F G; then A G shall be equal to G B, as the line C is to the line D.

PROBLEM II.

Given two lines A *and* B, *to find a third line in proportion to them.*—Fig. 2.

Make an angle E A B; let A E be equal to the line A, and A B equal to the line B—join E B, make E D equal to the line B—from D draw a line parallel

with E B, which will cut the line A B C at C, then shall B C be the line of proportion; for as A E : A B : : E D : B C.

PROBLEM III.

Given three lines A, B, C, *to find a fourth proportional line.*— Fig. 3.

With any two lines make an angle B A E, make A B equal to the line A, A E equal to the line B, and E D equal to the line C, join B E, and through D parallel to B E draw D C, cutting the line A B C at C, then shall B C be the fourth proportional line; for as A E : A B : : E D : B C,—that is, as B : A : : C : fourth proportional line.

PROBLEM IV.

If 180 *labourers do a piece of work in* 90 *days, in what time can* 100 *labourers do it?*—Fig. 3.

With the angle of the Fig. of last Prob. (or any other angle) let the line A B be equal to 100, the line A E = 90, and the line B C = 180, then will the line E D be equal to 162,—for A B : A E : : B C : E D, or 100 : 90 : : 180 : 162.

By these two last Problems, any proportion, either direct or inverse, can be calculated by lines or scales of equal parts.

PROBLEM V.

Between two lines E D = 36, *and* C D = 100, *to find a mean proportional line.*—Fig. 4.

Make a right line E C, equal to the two given lines—upon E C as a diameter, describe a semicircle E P C—upon D where the two given lines meet, raise the perpendicular line D P, cutting the arc in P, then D P shall be the mean geometrical proportional line required; for as C D : D P : : D P : D E, whence C D × D E = D P^2 = 3600, so D P = 60, as will be found by laying D P on the scale that measured E D and C D. By this Prob. II. may be solved.

PROBLEM VI.

To extract the square root of any number, suppose 3600.

RULE. A geometrical mean proportional between any two lines or numbers, is the square root of their product.

EXAMPLE.

100 × 36 = 3600, and a mean proportional between 100 and 36, is by the last problem = 60, therefore, 60 is the square root of 3600—also 10 × 360 = 3600, the mean proportional between these = 60, and 40 × 90 = 3600, and 30 × 120 = 3600, the mean proportional between 40 and 90, or between 30 and 120 is 60 = square root of 3600.

PROBLEM VII.

Between two given lines A *and* B, *to find two mean proportional lines.* Fig. 5.

Make a right angle K C H, drawing the side C H and C K at large, lay the line B, from C to E, and the line A, from C to D, and join E D, find its middle, which is at F, then with F E equal to F D, upon F describe the semicircle E G D with the lesser line B in the compasses, and one foot on D, cross the semicircle in G, upon the point G move a ruler till it cuts the lines C H and C K in H and K at an equal distance from F, then E H and D K shall be the two proportional lines sought, for as E C : D K : : D K : E H, and as E C : D K : : E H : C D.

PROBLEM VIII.

To extract the cube root.

RULE. The cube root of any number is the first of two mean proportionals between unity and that number; or which is the same, take any number which is less than the cube root of the given number, square this number, and divide the given number by it; then, the first of two mean proportionals between this quotient and the number taken, will be the cube root of the given number.

EXAMPLE.

What is the cube root of 67584? Take 32, which squared is 1024, then $\frac{67584}{1024} = 66$, and the first of two mean proportionals between 32 and 66, will be the cube root of 67584. Take Fig. 5 last Prob. and let the line A be equal to 66, and the line B equal to 32; then the line D K will be equal to 40.75 nearly, which is the cube root of 67584, being the first of two mean proportionals between 32 and 66. By this and Prob. VI. the cube and square root of any number can be easily found.

PROBLEM IX.

To find out two lines E F *and* F C, *which shall have such proportion to each other, as the square of a given line* A *hath to the square of another given line* B.—Fig. 6.

Make a right angle E D C, then lay the line B, from D to C, and A, from D to E, join C E. From D upon C E draw the perpendicular D F; then as $A^2 : B^2 :: EF : FC$.

PROBLEM X.

To divide a line C D *in power, as the line* A *is to the line* B. Fig 7.

Divide C D into such proportion as A to B, *i. e.* as B : A :: C E : E D; upon C D as a diameter, describe the semicircle C F D; upon E, raise the perpendicular

E F, cutting the semicircle in F; join F C and F D, which are the two lines required; for as B : A : : C F^2 : D F^2, and as A+B : C D^2 : : B : C F^2, and : : A : FD^2.

PROBLEM XI.

To enlarge any line C E *in power according to any proportion, suppose as the line* A *to the line* B. Fig. 7.

By Problem I., state it as A : B : : C E : C D, that is C B = A and C A = B, join E B, and through A, parallel to E B draw A D, cutting C E produced in D; upon C D describe the semicircle C F D; upon E erect the right angle E F, cutting the semicircle in F, join C F for the line required; for as A : B : : C E^2 : C F^2.

PROBLEM XII.

To cut a line A D *in extreme and mean proportions.* Fig. 8.

Upon A raise the perpendicular A F, making it equal to A D, bisect A F in G, and join G D, produce F A, making G I equal to G D, make A C equal to A I, which shall divide the line A D in the proportions as required; for A D : AC : : A C : C D.

Numb.	Square.	Cube.	Square Root.	Cube Root.
73	5329	389017	8.5440037	4.179339
74	5476	405224	8.6023253	4.198336
75	5625	421875	8.6602540	4.217163
76	5776	438976	8.7177979	4.235824
77	5929	456533	8.7749644	4.254321
78	6084	474552	8.8317609	4.272659
79	6241	493039	8.8881944	4.290841
80	6400	512000	8.9442719	4.308870
81	6561	531441	9.0000000	4.326749
82	6724	551368	9.0553851	4.344481
83	6889	571787	9.1104336	4.362071
84	7056	592704	9.1651514	4.379519
85	7225	614125	9.2195445	4.396830
86	7396	636056	9.2736185	4.414005
87	7569	658503	9.3273791	4.431047
88	7744	681472	9.3808315	4.447960
89	7921	704969	9.4339811	4.464745
90	8100	729000	9.4868330	4.481405
91	8281	753571	9.5393920	4.497942
92	8464	778688	9.5916630	4.514357
93	8649	804357	9.6436508	4.530655
94	8836	830584	9.6953597	4.546836
95	9025	857375	9.7467943	4.562903
96	9216	884736	9.7979590	4.578857
97	9409	912673	9.8488578	4.594701
98	9604	941192	9.8994949	4.610436
99	9801	970299	9.9498744	4.626065
100	10000	1000000	10.0000000	4.641589
101	10201	1030301	10.0498756	4.657010
102	10404	1061208	10.0995049	4.672330
103	10609	1092727	10.1488916	4.687548
104	10816	1124864	10.1980390	4.702669
105	11025	1157625	10.2469508	4.717694
106	11236	1191016	10.2956301	4.732624
107	11449	1225043	10.3440804	4.747459
108	11664	1259712	10.3923048	4.762203
109	11881	1295029	10.4403065	4.776856
110	12100	1331000	10.4880885	4.791420
111	12321	1367631	10.5356538	4.805896

Numb.	Square.	Cube.	Square Root.	Cube Root.
112	12544	1404928	10.5830052	4.820284
113	12769	1442897	10.6301458	4.834588
114	12996	1481544	10.6770783	4.848808
115	13225	1520875	10.7238053	4.862944
116	13456	1560896	10.7703296	4.876999
117	13689	1601613	10.8166538	4.890973
118	13924	1643032	10.8627805	4.904868
119	14161	1685159	10.9087121	4.918685
120	14400	1728000	10.9544512	4.932424
121	14641	1771561	11.0000000	4.946088
122	14884	1815848	11.0453610	4.959675
123	15129	1860867	11.0905365	4.973190
124	15376	1906624	11.1355287	4.986631
125	15625	1953125	11.1803399	5.000000
126	15876	2000376	11.2249722	5.013298
127	16129	2048383	11.2694277	5.026526
128	16384	2097152	11.3137085	5.039684
129	16641	2146689	11.3578167	5.052774
130	16900	2197000	11.4017543	5.065797
131	17161	2248091	11.4455231	5.078753
132	17424	2299968	11.4891253	5.091643
133	17689	2352637	11.5325626	5.104469
134	17956	2406104	11.5758369	5.117230
135	18225	2460375	11.6189500	5.129928
136	18496	2515456	11.6619038	5.142563
137	18769	2571353	11.7046999	5.155137
138	19044	2628072	11.7473444	5.167649
139	19321	2685619	11.7898261	5.180101
140	19600	2744000	11.8321596	5.192494
141	19881	2803221	11.8743421	5.204828
142	20164	2863288	11.9163753	5.217103
143	20449	2924207	11.9582607	5.229321
144	20736	2985984	12.0000000	5.241482
145	21025	3048625	12.0415946	5.253588
146	21316	3112136	12.0830460	5.265637
147	21609	3176523	12.1243557	5.277632
148	21904	3241792	12.1655251	5.289572
149	22201	3307949	12.2065556	5.301459
150	22500	3375000	12.2474487	5.313293

Numb.	Square.	Cube.	Square Root.	Cube Root.
151	22801	3442951	12.2882057	5.325074
152	23104	3511808	12.3288280	5.336803
153	23409	3581577	12.3693169	5.348481
154	23716	3652264	12.4096736	5.360108
155	24025	3723875	12.4498996	5.371685
156	24336	3796416	12.4899960	5.383231
157	24649	3869893	12.5299641	5.394690
158	24964	3944312	12.5698051	5.406120
159	25281	4019679	12.6095202	5.417501
160	25600	4096000	12.6491106	5.428835
161	25921	4173281	12.6885775	5.440122
162	26244	4251528	12.7279221	5.451362
163	26569	4330747	12.7671453	5.462556
164	26896	4410944	12.8062485	5.473703
165	27225	4492125	12.8452326	5.484806
166	27556	4574296	12.8840987	5.495865
167	27889	4657463	12.9228480	5.506879
168	28224	4741632	12.9614814	5.517848
169	28561	4826809	13.0000000	5.528775
170	28900	4913000	13.0384048	5.539658
171	29241	5000211	13.0766968	5.550499
172	29584	5088448	13.1148770	5.561298
173	29929	5177717	13.1529464	5.572054
174	30276	5268024	13.1909060	5.582770
175	30625	5359375	13.2287566	5.593445
176	30976	5451776	13.2664992	5.604079
177	31329	5545233	13.3041347	5.614673
178	31684	5639752	13.3416641	5.625226
179	32041	5735339	13.3790882	5.635741
180	32400	5832000	13.4164079	5.646216
181	32761	5929741	13.4536240	5.656652
182	33124	6028568	13.4907376	5.667051
183	33489	6128487	13.5277493	5.677411
184	33856	6229504	13.5646600	5.687734
185	34225	6331625	13.6014705	5.698019
186	34596	6434856	13.6381817	5.708267
187	34969	6539203	13.6747943	5.718479
188	35344	6644672	13.7113092	5.728654
189	35721	6751269	13.7477271	5.738794

Numb.	Square.	Cube.	Square Root.	Cube Root.
190	36100	6859000	13.7840488	5.748897
191	36481	6967871	13.8202750	5.758965
192	36864	7077888	13.8564065	5.768998
193	37249	7189057	13.8924440	5.778996
194	37636	7301384	13.9283883	5.788960
195	38025	7414875	13.9642400	5.798890
196	38416	7529536	14.0000000	5.808786
197	38809	7645373	14.0356688	5.818648
198	39204	7762392	14.0712473	5.828476
199	39601	7880599	14.1067360	5.838272
200	40000	8000000	14.1421356	5.848085
201	40401	8120601	14.1774469	5.857765
202	40804	8242408	14.2126704	5.867464
203	41209	8365427	14.2478068	5.877130
204	41616	8489664	14.2828569	5.886765
205	42025	8615125	14.3178211	5.896368
206	42436	8741816	14.3527001	5.905941
207	42849	8869743	14.3874946	5.915481
208	43264	8998912	14.4222051	5.924991
209	43681	9123329	14.4568323	5.934473
210	44100	9261000	14.4913767	5.943911
211	44521	9393931	14.5258390	5.953341
212	44944	9528128	14.5602198	5.962731
213	45369	9663597	14.5945195	5.972091
214	45796	9800344	14.6287388	5.981426
215	46225	9938375	14.6628783	5.990727
216	46656	10077696	14.6969385	6.000000
217	47089	10218313	14.7309199	6.009244
218	47524	10360232	14.7648231	6.018463
219	47961	10503459	14.7986486	6.027650
220	48400	10648000	14.8323970	6.036811
221	48841	10793861	14.8660687	6.045943
222	49284	10941048	14.8996644	6.055048
223	49729	11089567	14.9331845	6.064126
224	50176	11239424	14.9666295	6.073177
225	50625	11390625	15.0000000	6.082201
226	51076	11543176	15.0332964	6.091199
227	51529	11697083	15.0665192	6.100170
228	51984	11852352	15.0996689	6.109115

15

Numb.	Square.	Cube.	Square Root.	Cube Root.
229	52441	12008989	15.1327460	6.118032
230	52900	12167000	15.1657509	6.126925
231	53361	12326391	15.1986842	6.135792
232	53824	12487168	15.2315462	6.144634
233	54289	12649337	15.2643375	6.153449
234	54756	12812904	15.2970585	6.162239
235	55225	12977875	15.3297097	6.171005
236	55696	13144256	15.3622915	6.179747
237	56169	13312053	15.3948043	6.188463
238	56644	13481272	15.4272486	6.197154
239	57121	13651919	15.4596248	6.205821
240	57600	13824000	15.4919334	6.214464
241	58081	13997521	15.5241747	6.223083
242	58564	14172488	15.5563492	6.231678
243	59049	14348907	15.5884573	6.240251
244	59536	14526784	15.6204994	6.248800
245	60025	14706125	15.6524758	6.257324
246	60516	14886936	15.6843871	6.265826
247	61009	15069223	15.7162336	6.274304
248	61504	15252992	15.7480157	6.282760
249	62001	15438249	15.7797338	6.291194
250	62500	15625000	15.8113883	6.299604
251	63001	15813251	15.8429795	6.307992
252	63504	16003008	15.8745079	6.316359
253	64009	16194277	15.9059737	6.324704
254	64516	16387064	15.9373775	6.333025
255	65025	16581375	15.9687194	6.341325
256	65536	16777216	16.0000000	6.349602
257	66049	16974593	16.0312195	6.357859
258	66564	17173512	16.0623784	6.366095
259	67081	17373979	16.0934769	6.374310
260	67600	17576000	16.1245155	6.382504
261	68121	17779581	16.1554944	6.390676
262	68644	17984728	16.1864141	6.398827
263	69169	18191447	16.2172747	6.406958
264	69696	18399744	16.2480768	6.415068
265	70225	18609625	16.2788206	6.423157
266	70756	18821096	16.3095064	6.431226
267	71289	19034163	16.3401346	6.439275

Numb.	Square.	Cube.	Square Root.	Cube Root.
268	71824	19248832	16.3707055	6.447305
269	72361	19465109	16.4012195	6.455314
270	72900	19683000	16.4316767	6.463304
271	73441	19902511	16.4620776	6.471274
272	73984	20123648	16.4924225	6.479224
273	74529	20346417	16.5227116	6.487153
274	75076	20570824	16.5529454	6.495064
275	75625	20796875	16.5831240	6.502956
276	76176	21024576	16.6132477	6.510829
277	76729	21253933	16.6433170	6.518684
278	77284	21484952	16.6733320	6.526519
279	77841	21717639	16.7032931	6.534335
280	78400	21952000	16.7332005	6.542132
281	78961	22188041	16.7630546	6.549911
282	79524	22425768	16.7928556	6.557672
283	80089	22665187	16.8226038	6.565415
284	80656	22906304	16.8522995	6.573139
285	81225	23149125	16.8819430	6.580844
286	81796	23393656	16.9115345	6.588531
287	82369	23639903	16.9410743	6.596202
288	82944	23887872	16.9705627	6.603854
289	83521	24137569	17.0000000	6.611488
290	84100	24389000	17.0293864	6.619106
291	84681	24642171	17.0587221	6.626705
292	85264	24897088	17.0880075	6.634287
293	85849	25153757	17.1172428	6.641851
294	86436	25412184	17.1464282	6.649399
295	87025	25672375	17.1755640	6.656930
296	87616	25934336	17.2046505	6.664443
297	88209	26198073	17.2336879	6.671940
298	88804	26463592	17.2626762	6.679419
299	89401	26730899	17.2916165	6.686882
300	90000	27000000	17.3205081	6.694328
301	90601	27270901	17.3493516	6.701758
302	91204	27543608	17.3781472	6.709172
303	91809	27818127	17.4068952	6.716569
304	92416	28094464	17.4355958	6.723950
305	93025	28372625	17.4642492	6.731316
306	93636	28652616	17.4928557	6.738665

Numb.	Square.	Cube.	Square Root.	Cube Root.
307	94249	28934443	17.5214155	6.745997
308	94864	29218112	17.5499288	6.753313
309	95481	29503629	17.5783958	6.760614
310	96100	29791000	17.6068169	6.767899
311	96721	30080231	17.6351921	6.775168
312	97344	30371328	17.6635217	6.782422
313	97969	30664297	17.6918060	6.789661
314	98596	30959144	17.7200451	6.796884
315	99225	31255875	17.7482393	6.804091
316	99856	31554496	17.7763888	6.811284
317	100489	31855013	17.8044938	6.818461
318	101124	32157432	17.8325545	6.825624
319	101761	32461759	17.8605711	6.832771
320	102400	32768000	17.8885438	6.839903
321	103041	33076161	17.9164729	6.847021
322	103684	33386248	17.9443584	6.854124
323	104329	33698267	17.9722008	6.861211
324	104976	34012224	18.0000000	6.868284
325	105625	34328125	18.0277564	6.875343
326	106276	34645976	18.0554701	6.882388
327	106929	34965783	18.0831413	6.889419
328	107584	35287552	18.1107703	6.896435
329	108241	35611289	18.1383571	6.903436
330	108900	35937000	18.1659021	6.910423
331	109561	36264691	18.1934054	6.917396
332	110224	36594368	18.2208672	6.924355
333	110889	36926037	18.2482876	6.931300
334	111556	37259704	18.2756669	6.938232
335	112225	37595375	18.3030052	6.945149
336	112896	37933056	18.3303028	6.952053
337	113569	38272753	18.3575598	6.958943
338	114244	38614472	18.3847763	6.965819
339	114921	38958219	18.4119526	6.972682
340	115600	39304000	18.4390889	6.979532
341	116281	39651821	18.4661853	6.986369
342	116964	40001688	18.4932420	6.993491
343	117649	40353607	18.5202592	7.000000
344	118336	40707584	18.5472370	7.006796
345	119025	41063625	18.5741756	7.013579

Numb.	Square.	Cube.	Square Root.	Cube Root.
346	119716	41421736	18.6010752	7.020349
347	120409	41781923	18.6279360	7.027106
348	121104	42144192	18.6547581	7.033850
349	121801	42508549	18.6815417	7.040581
350	122500	42875000	18.7082869	7.047208
351	123201	43243551	18.7349940	7.054003
352	123904	43614208	18.7616630	7.060696
353	124609	43986977	18.7882942	7.067376
354	125316	44361864	18.8148877	7.074043
355	126025	44738875	18.8414437	7.080698
356	126736	45118016	18.8679623	7.087341
357	127449	45499293	18.8944436	7.093970
358	128164	45882712	18.9208879	7.100588
359	128881	46268279	18.9472953	7.107193
360	129600	46656000	18.9736660	7.113786
361	130321	47045881	19.0000000	7.120367
362	131044	47437928	19.0262976	7.126935
363	131769	47832147	19.0525589	7.133492
364	132496	48228544	19.0787840	7.140037
365	133225	48627125	19.1049732	7.146569
366	133956	49027896	19.1311265	7.153090
367	134689	49430863	19.1572441	7.159599
368	135424	49836032	19.1833261	7.166095
369	136161	50243409	19.2093727	7.172580
370	136900	50653000	19.2353841	7.179054
371	137641	51064811	19.2613603	7.185516
372	138384	51478848	19.2873015	7.191966
373	139129	51895117	19.3132079	7.198405
374	139876	52313624	19.3390796	7.204832
375	140625	52734375	19.3649167	7.211247
376	141376	53157376	19.3907194	7.217652
377	142129	53582633	19.4164878	7.224045
378	142884	54010152	19.4422221	7.230427
379	143641	54439939	19.4679223	7.236797
380	144400	54872000	19.4935887	7.243156
381	145161	55306341	19.5192213	7.249504
382	145924	55742968	19.5448203	7.255841
383	146689	56181887	19.5703858	7.262167
384	147456	56623104	19.5959179	7.268482

Numb.	Square.	Cube.	Square Root.	Cube Root.
385	148225	57066625	19.6214169	7.274786
386	148996	57512456	19.6468827	7.281079
387	149769	57960603	19.6723156	7.287362
388	150544	58411072	19.6977156	7.293633
389	151321	58863869	19.7230829	7.299893
390	152100	59319000	19.7484177	7.306143
391	152881	59776471	19.7737199	7.312383
392	153664	60236288	19.7989899	7.318611
393	154449	60698457	19.8242276	7.324829
394	155236	61162984	19.8494332	7.331037
395	156025	61629875	19.8746069	7.337234
396	156816	62099136	19.8997487	7.343420
397	157609	62570773	19.9248588	7.349596
398	158404	63044792	19.9499373	7.355762
399	159201	63521199	19.9749844	7.361917
400	160000	64000000	20.0000000	7.368063
401	160801	64481201	20.0249844	7.374198
402	161604	64964808	20.0499377	7.380322
403	162409	65450827	20.0748599	7.386437
404	163216	65939264	20.0997512	7.392542
405	164025	66430125	20.1246118	7.398636
406	164836	66923416	20.1494417	7.404720
407	165649	67419143	20.1742410	7.410794
408	166464	67911312	20.1990099	7.416859
409	167281	68417929	20.2237484	7.422914
410	168100	68921000	20.2484567	7.428958
411	168921	69426531	20.2731349	7.434993
412	169744	69934528	20.2977831	7.441018
413	170569	70444997	20.3224014	7.447033
414	171396	70957944	20.3469899	7.453039
415	172225	71473375	20.3715488	7.459036
416	173056	71991296	20.3960781	7.465022
417	173889	72511713	20.4205779	7.470999
418	174724	73034632	20.4450483	7.476966
419	175561	73560059	20.4694895	7.482924
420	176400	74088000	20.4939015	7.488872
421	177241	74618461	20.5182845	7.494810
422	178084	75151448	20.5426386	7.500740
423	178929	75686967	20.5669638	7.506660

Numb.	Square.	Cube.	Square Root.	Cube Root.
424	179776	76225024	20.5912603	7.512571
425	180625	76765625	20.6155281	7.518473
426	181476	77308776	20.6397674	7.524365
427	182329	77854483	20.6639783	7.530248
428	183184	78402752	20.6881609	7.536121
429	184041	78953589	20.7123152	7.541986
430	184900	79507000	20.7364414	7.547841
431	185761	80062991	20.7605395	7.553688
432	186624	80621568	20.7846097	7.559525
433	187489	81182737	20.8086520	7.565353
434	188356	81746504	20.8326667	7.571173
435	189225	82312875	20.8566536	7.576984
436	190096	82881856	20.8806130	7.582786
437	190969	83453453	20.9045450	7.588579
438	191844	84027672	20.9284495	7.594363
439	192721	84604519	20.9523268	7.600138
440	193600	85184000	20.9761770	7.605905
441	194481	85766121	21.0000000	7.611662
442	195364	86350388	21.0237960	7.617411
443	196249	86938307	21.0475652	7.623151
444	197136	87528384	21.0713075	7.628883
445	198025	88121125	21.0950231	7.634606
446	198916	88716536	21.1187121	7.640321
447	199809	89314623	21.1423745	7.646027
448	200704	89915392	21.1660105	7.651725
449	201601	90518849	21.1896201	7.657414
450	202500	91125000	21.2132034	7.663094
451	203401	91733851	21.2367606	7.668766
452	204304	92345408	21.2602916	7.674430
453	205209	92959677	21.2837967	7.680085
454	206106	93576664	21.3072758	7.685732
455	207025	94196375	21.3307290	7.691371
456	207936	94818816	21.3541565	7.697002
457	208849	95443993	21.3775583	7.702624
458	209764	96071912	21.4009346	7.708238
459	210681	96702579	21.4242853	7.713844
460	211600	97336000	21.4476106	7.719442
461	212521	97972181	21.4709106	7.725032
462	213444	98611128	21.4941853	7.730614

Numb.	Square.	Cube.	Square Root.	Cube Root.
463	214369	99252847	21.5174348	7.736187
464	215296	99897344	21.5406592	7.741753
465	216225	100544625	21.5638587	7.747310
466	217156	101194696	21.5870331	7.752860
467	218089	101847563	21.6101828	7.758402
468	219024	102503232	21.6333077	7.763936
469	219961	103161709	21.6564078	7.769462
470	220900	103823000	21.6794834	7.774980
471	221841	104487111	21.7025344	7.780490
472	222784	105154048	21.7255610	7.785992
473	223729	105823817	21.7485632	7.791487
474	224676	106496424	21.7715411	7.796974
475	225625	107171875	21.7944947	7.802453
476	226576	107850176	21.8174242	7.807925
477	227529	108531333	21.8403297	7.813389
478	228484	109215352	21.8632111	7.818845
479	229441	109902239	21.8860686	7.824294
480	230400	110592000	21.9089023	7.829735
481	231361	111284641	21.9317122	7.835168
482	232324	111980168	21.9544984	7.840594
483	233289	112678587	21.9772610	7.846013
484	234256	113379904	22.0000000	7.851424
485	235225	114084125	22.0227155	7.856828
486	236196	114791256	22.0454077	7.862224
487	237169	115501303	22.0680765	7.867613
488	238144	116214272	22.0907220	7.872994
489	239121	116930169	22.1133444	7.878368
490	240100	117649000	22.1359436	7.883734
491	241081	118370771	22.1585198	7.889094
492	242064	119095488	22.1810730	7.894446
493	243049	119823157	22.2036033	7.899791
494	244036	120553784	22.2261108	7.905129
495	245025	121287375	22.2485955	7.910460
496	246016	122023936	22.2710575	7.915784
497	247009	122763473	22.2934968	7.921100
498	248004	123505992	22.3159136	7.926408
499	249001	124251499	22.3383079	7.931710
500	250000	125000000	22.3606798	7.937005
501	251001	125751501	22.3830293	7.942293

Numb.	Square.	Cube.	Square Root.	Cube Root.
502	252004	126506008	22.4053565	7.947573
503	253009	127263527	22.4276615	7.952847
504	254016	128024064	22.4499443	7.958114
505	255025	128787625	22.4722051	7.963374
506	256036	129554216	22.4944438	7.968627
507	257049	130323843	22.5166605	7.973873
508	258064	131096512	22.5388553	7.979112
509	259081	131872229	22.5610283	7.984344
510	260100	132651000	22.5831796	7.989569
511	261121	133432831	22.6053091	7.994788
512	262144	134217728	22.6274170	8.000000
513	263169	135005697	22.6495033	8.005205
514	264196	135796744	22.6715681	8.010403
515	265225	136590875	22.6936114	8.015595
516	266256	137388096	22.7156334	8.020779
517	267289	138188413	22.7376340	8.025957
518	268324	138991832	22.7596134	8.031129
519	269361	139798359	22.7815715	8.036293
520	270400	140608000	22.8035085	8.041451
521	271441	141420761	22.8254244	8.046603
522	272484	142236648	22.8473193	8.051748
523	273529	143055667	22.8691933	8.056886
524	274576	143877824	22.8910463	8.062018
525	275625	144703125	22.9128785	8.067143
526	276676	145531576	22.9346899	8.072262
527	277729	146363183	22.9564806	8.077374
528	278784	147197952	22.9782506	8.082480
529	279841	148035889	23.0000000	8.087579
530	280900	148877000	23.0217289	8.092672
531	281961	149721291	23.0434372	8.097758
532	283024	150568768	23.0651252	8.102838
533	284089	151419437	23.0867928	8.107912
534	285156	152273304	23.1084400	8.112980
535	286225	153130375	23.1300670	8.118041
536	287296	153990656	23.1516738	8.123096
537	288369	154854153	23.1732605	8.128144
538	289444	155720872	23.1948270	8.133186
539	290521	156590819	23.2163735	8.138223
540	291600	157464000	23.2379001	8.143253

Numb.	Square.	Cube.	Square Root.	Cube Root.
541	292681	158340421	23.2594067	8.148276
542	293764	159220088	23.2808935	8.153293
543	294849	160103007	23.3023604	8.158304
544	295936	160989184	23.3238076	8.163309
545	297025	161878625	23.3452351	8.168308
546	298116	162771336	23.3666429	8.173302
547	299209	163667323	23.3880311	8.178289
548	300304	164566592	23.4093998	8.183269
549	301401	165469149	23.4307490	8.188244
550	302500	166375000	23.4520788	8.193212
551	303601	167284151	23.4733892	8.198175
552	304704	168196608	23.4946802	8.203131
553	305809	169112377	23.5159520	8.208082
554	306916	170031464	23.5372046	8.213027
555	308025	170953875	23.5584380	8.217965
556	309136	171879616	23.5796522	8.222898
557	310249	172808693	23.6008474	8.227825
558	311364	173741112	23.6220236	8.232746
559	312481	174676879	23.6431808	8.237661
560	313600	175616000	23.6643191	8.242570
561	314721	176558481	23.6854386	8.247474
562	315844	177504328	23.7065392	8.252371
563	316969	178453547	23.7276210	8.257263
564	318096	179406144	23.7486842	8.262149
565	319225	180362125	23.7697286	8.267029
566	320356	181321496	23.7907545	8.271903
567	321489	182284263	23.8117618	8.276772
568	322624	183250432	23.8327506	8.281635
569	323761	184220009	23.8537209	8.286493
570	324900	185193000	23.8746728	8.291344
571	326041	186169411	23.8956063	8.296190
572	327184	187149248	23.9165215	8.301030
573	328329	188132517	23.9374184	8.305865
574	329476	189119224	23.9582971	8.310694
575	330625	190109375	23.9791576	8.315517
576	331776	191102976	24.0000000	8.320335
577	332929	192100033	24.0208243	8.325147
578	334084	193100552	24.0416306	8.329954
579	335241	194104539	24.0624188	8.334755

Numb.	Square.	Cube.	Square Root.	Cube Root.
580	336400	195112000	24.0831892	8.339551
581	337561	196122941	24.1039416	8.344341
582	338724	197137368	24.1246762	8.349125
583	339889	198155287	24.1453929	8.353904
584	341056	199176704	24.1660919	8.358678
585	342225	200201625	24.1867732	8.363446
586	343396	201230056	24.2074369	8.368209
587	344569	202262003	24.2280829	8.372966
588	345744	203297472	24.2487113	8.377718
589	346921	204336469	24.2693222	8.382465
590	348100	205379000	24.2899156	8.387206
591	349281	206425071	24.3104916	8.391942
592	350464	207474688	24.3310501	8.396673
593	351649	208527857	24.3515913	8.401398
594	352836	209584584	24.3721152	8.406118
595	354025	210644875	24.3926218	8.410832
596	355216	211708736	24.4131112	8.415541
597	356409	212776173	24.4335834	8.420245
598	357604	213847192	24.4540385	8.424944
599	358801	214921799	24.4744765	8.429638
600	360000	216000000	24.4948974	8.434327
601	361201	217081801	24.5153013	8.439009
602	362404	218167208	24.5356883	8.443687
603	363609	219256227	24.5560583	8.448360
604	364816	220348864	24.5764115	8.453027
605	366025	221445125	24.5967478	8.457689
606	367236	222545016	24.6170673	8.462347
607	368449	223648543	24.6373700	8.466999
608	369664	224755712	24.6576560	8.471647
609	370881	225866529	24.6779254	8.476289
610	372100	226981000	24.6981781	8.480926
611	373321	228099131	24.7184142	8.485557
612	374544	229220928	24.7386338	8.490184
613	375769	230346397	24.7588368	8.494806
614	376996	231475544	24.7790234	8.499423
615	378225	232608375	24.7991935	8.504034
616	379456	233744896	24.8193473	8.508641
617	380689	234885113	24.8394847	8.513243
618	381924	236029032	24.8596058	8.517840

Numb.	Square	Cube.	Square Root.	Cube Root.
619	383161	237176659	24.8797106	8.522432
620	384400	238328000	24.8997992	8.527018
621	385641	239483061	24.9198716	8.531600
622	386884	240641848	24.9399278	8.536177
623	388129	241804367	24.9599679	8.540749
624	389376	242970624	24.9799920	8.545317
625	390625	244140625	25.0000000	8.549879
626	391876	245314376	25.0199920	8.554437
627	393129	246491883	25.0399681	8.558990
628	394384	247673152	25.0599282	8.563537
629	395641	248858189	25.0798724	8.568080
630	396900	250047000	25.0998008	8 572618
631	398161	251239591	25.1197134	8.577152
632	399424	252435968	25.1396102	8.581680
633	400689	253636137	25.1594913	8.586204
634	401956	254840104	25.1793566	8.590723
635	403225	256047875	25.1992063	8.595238
636	404496	257259456	25.2190404	8.599747
637	405769	258474853	25.2388589	8.604252
638	407044	259694072	25.2586619	8.608752
639	408321	260917119	25.2784493	8.613248
640	409600	262144000	25.2982213	8.617738
641	410881	263374721	25.3179778	8.622224
642	412164	264609288	25.3377189	8.626706
643	413449	265847707	25.3574447	8.631183
644	414736	267089984	25.3771551	8.635655
645	416025	268336125	25.3968502	8.640122
646	417316	269586136	25.4165301	8.644585
647	418609	270840023	25.4361947	8.649043
648	419904	272097792	25.4558441	8.653497
649	421201	273359449	25.4754784	8.657946
650	422500	274625000	25.4950076	8.662301
651	423801	275894451	25.5147016	8.666831
652	425104	277167808	25.5342907	8.671266
653	426409	278445077	25.5538647	8.675697
654	427716	279726264	25.5734237	8.680123
655	429025	281011375	25.5929678	8.684545
656	430336	282300416	25.6124969	8.688963
657	431649	283593393	25.6320112	8.693376

Numb.	Square.	Cube.	Square Root.	Cube Root.
658	432964	284890312	25.6515107	8.697784
659	434281	286191179	25.6709953	8.702188
660	435600	287496000	25.6904652	8.706587
661	436921	288804781	25.7099203	8.710982
662	438244	290117528	25.7203607	8.715373
663	439569	291434247	25.7487864	8.719759
664	440896	292754944	25.7681975	8.724141
665	442225	294079625	25.7875939	8.728518
666	443556	295408296	25.8069758	8.732891
667	444889	296740963	25.8263431	8.737260
668	446224	298077632	25.8456960	8.741624
669	447561	299418309	25.8650343	8.745984
670	448900	300763000	25.8843582	8.750340
671	450241	302111711	25.9036677	8.754691
672	451584	303464448	25.9229628	8.759038
673	452929	304821217	25.9422435	8.763380
674	454276	306182024	25.9615100	8.767719
675	455625	307546875	25.9807621	8.772053
676	456976	308915776	26.0000000	8.776382
677	458329	310288733	26.0192237	8.780708
678	459684	311665752	26.0384331	8.785029
679	461041	313046839	26.0576284	8.789346
680	462400	314432000	26.0768096	8.793659
681	463761	315821241	26.0959767	8.797967
682	465124	317214568	26.1151297	8.802272
683	466489	318611987	26.1342687	8.806572
684	467856	320013504	26.1533937	8.810868
685	469225	321419125	26.1725047	8.815159
686	470596	322828856	26.1916017	8.819447
687	471969	324242703	26.2106848	8.823730
688	473344	325660672	26.2297541	8.828009
689	474721	327082769	26.2488095	8.832285
690	476100	328509000	26.2678511	8.836556
691	477481	329939371	26.2868789	8.840822
692	478864	331373888	26.3058929	8.845085
693	480249	332812557	26.3248932	8.849344
694	481636	334255384	26.3438797	8.853598
695	483025	335702375	26.3628527	8.857849
696	484416	337153536	26.3818119	8.862095

Numb.	Square.	Cube.	Square Root.	Cube Root.
697	485809	338608873	26.4007576	8.866337
698	487204	340068392	26.4196896	8.870575
699	488601	341532099	26.4386081	8.874809
700	490000	343000000	26.4575131	8.879040
701	491401	344472101	26.4764046	8.883266
702	492804	345948088	26.4952826	8.887488
703	494209	347428927	26.5141472	8.891706
704	495616	348913664	26.5329988	8.895920
705	497025	350402625	26.5518361	8.900130
706	498436	351895816	26.5706605	8.904336
707	499849	353393243	26.5894716	8.908538
708	501264	354894912	26.6082694	8.912736
709	502681	356400829	26.6270539	8.916931
710	504100	357911000	26.6458252	8.921121
711	505521	359425431	26.6645833	8.925307
712	506944	360944128	26.6833281	8.929490
713	508369	362467097	26.7020598	8.933668
714	509796	363994344	26.7207784	8.937843
715	511225	365525875	26.7394839	8.942014
716	512656	367061696	26.7581763	8.946180
717	514089	368601813	26.7768557	8.950343
718	515524	370146232	26.7955220	8.954502
719	516961	371694959	26.8141754	8.958658
720	518400	373248000	26.8328157	8.962809
721	519841	374805361	26.8514432	8.966957
722	521284	376367048	26.8700577	8.971100
723	522729	377933067	26.8886593	8.975240
724	524176	379503424	26.9072481	8.979376
725	525625	381078125	26.9258240	8.983508
726	527076	382657176	26.9443872	8.987637
727	528529	384240583	26.9629375	8.991762
728	529984	385828352	26.9814751	8.995883
729	531441	387420489	27.0000000	9.000000
730	532900	389017000	27.0185122	9.004113
731	534361	390617891	27.0370117	9.008222
732	535824	392223168	27.0554985	9.012328
733	537289	393832837	27.0739727	9.016430
734	538756	395446904	27.0924344	9.020529
735	540225	397065375	27.1108834	9.024623

Numb.	Square.	Cube.	Square Root.	Cube Root.
736	541696	398688256	27.1293199	9.028714
737	543169	400315553	27.1477439	9.032802
738	544644	401947272	27.1661554	9.036885
739	546121	403583419	27.1845544	9.040965
740	547600	405224000	27.2029410	9.045041
741	549081	406869021	27.2213152	9.049114
742	550564	408518488	27.2396769	9.053183
743	552049	410172407	27.2580263	9.057248
744	553536	411830784	27.2763634	9.061309
745	555025	413493625	27.2946881	9.065367
746	556516	415160936	27.3130006	9.069422
747	558009	416832723	27.3313007	9.073472
748	559504	418508992	27.3495887	9.077519
749	561001	420189749	27.3678644	9.081563
750	562500	421875000	27.3861279	9.085603
751	564001	423564751	27.4043792	9.089639
752	565504	425259008	27.4226184	9.093672
753	567009	426957777	27.4408455	9.097701
754	568516	428661064	27.4590604	9.101726
755	570025	430368875	27.4772633	9.105748
756	571536	432081216	27.4954542	9.109766
757	573049	433798093	27.5136330	9.113781
758	574564	435519512	27.5317998	9.117793
759	576081	437245479	27.5499546	9.121801
760	577600	438976000	27.5680975	9.125805
761	579121	440711081	27.5862284	9.129806
762	580644	442450728	27.6043475	9.133803
763	582169	444194947	27.6224546	9.137797
764	583696	445943744	27.6405499	9.141788
765	585225	447697125	27.6586334	9.145774
766	586756	449455096	27.6767050	9.149757
767	588289	451217663	27.6947648	9.153737
768	589824	452984832	27.7128129	9.157713
769	591361	454756609	27.7308492	9.161686
770	592900	456533000	27.7488739	9.165656
771	594441	458314011	27.7668868	9.169622
772	595984	460099648	27.7848880	9.173585
773	597529	461889917	27.8028775	9.177544
774	599076	463684824	27.8208555	9.181500

Numb.	Square.	Cube.	Square Root.	Cube Root.
775	600625	465484375	27.8388218	9.185452
776	602176	467288576	27.8567766	9.189401
777	603729	469097433	27.8747197	9.193347
778	605284	470910952	27.8926514	9.197289
779	606841	472729139	27.9105715	9.201228
780	608400	474552000	27.9284801	9.205164
781	609961	476379541	27.9463772	9.209096
782	611524	478211768	27.9642629	9.213025
783	613089	480048687	27.9821372	9.216950
784	614656	481890304	28.0000000	9.220872
785	616225	483736025	28.0178515	9.224791
786	617796	485587656	28.0356915	9.228706
787	619369	487443403	28.0535203	9.232618
788	620944	489303872	28.0713377	9.237527
789	622521	491169069	28.0891438	9.240433
790	624100	493039000	28.1069386	9.244335
791	625681	494913671	28.1247222	9.248234
792	627264	496793088	28.1424946	9.252130
793	628849	498677257	28.1602557	9.256022
794	630436	500566184	28.1780056	9.259911
795	632025	502459875	28.1957444	9.263797
796	633616	504358336	28.2134720	9.267679
797	635209	506261573	28.2311884	9.271559
798	636804	508169592	28.2488938	9.275435
799	638401	510082399	28.2665881	9.279308
800	640000	512000000	28.2842712	9.283177
801	641601	513922401	28.3019434	9.287044
802	643204	515849608	28.3196045	9.290907
803	644809	517781627	28.3372546	9.294767
804	646416	519718464	28.3548938	9.298623
805	648025	521660125	28.3725219	9.302477
806	649636	523606616	28.3901391	9.306327
807	651249	525557943	28.4077454	9.310175
808	652864	527514112	28.4253408	9.314019
809	654481	529475129	28.4429253	9.317859
810	656100	531441000	28.4604989	9.321697
811	657721	533411731	28.4780617	9.325532
812	659344	535387328	28.4956137	9.329363
813	660969	537366797	28.5131549	9.333191

Numb.	Square.	Cube.	Square Root.	Cube Root.
814	662596	539353144	28.5306852	9.337016
815	664225	541343375	28.5482048	9.340838
816	665856	543338496	28.5657137	9.344657
817	667489	545338513	28.5832119	9.348473
818	669124	547343432	28.6006993	9.352285
819	670761	549353259	28.6181760	9.356095
820	672400	551368000	28.6356421	9.359901
821	674041	553387661	28.6530976	9.363704
822	675684	555412248	28.6705424	9.367505
823	677329	557441767	28.6879766	9.371302
824	678976	559476224	28.7054002	9.375096
825	680625	561515625	28.7228132	9.378887
826	682276	563559976	28.7402157	9.382675
827	683929	565609283	28.7576077	9.386460
828	685584	567663552	28.7749891	9.390241
829	687241	569722789	28.7923601	9.394020
830	688900	571787000	28.8097206	9.397796
831	690561	573856191	28.8270706	9.401569
832	692224	575930368	28.8444102	9.405338
833	693889	578009537	28.8617394	9.409105
834	695556	580093704	28.8790582	9.412869
835	697225	582182875	28.8963666	9.416630
836	698896	584277056	28.9136646	9.420387
837	700569	586376253	28.9309523	9.424141
838	702244	588480472	28.9482297	9.427893
839	703921	590589719	28.9654967	9.431642
840	705600	592704000	28.9827535	9.435388
841	707281	594823321	29.0000000	9.439130
842	708964	596947688	29.0172363	9.442870
843	710649	599077107	29.0344623	9.446607
844	712336	601211584	29.0516781	9.450341
845	714025	603351125	29.0688837	9.454071
846	715716	605495736	29.0860791	9.457799
847	717409	607645423	29.1032644	9.461524
848	719104	609800192	29.1204396	9.465247
849	720801	611960049	29.1376046	9.468966
850	722500	614125000	29.1547595	9.472682
851	724201	616295051	29.1719043	9.476395
852	725904	618470208	29.1890390	9.480106

Numb.	Square.	Cube.	Square Root.	Cube Root.
853	727609	620650477	29.2061637	9.483813
854	729316	622835864	29.2232784	9.487518
855	731025	625026375	29.2403830	9.491219
856	732736	627222016	29.2574777	9.494918
857	734449	629422793	29.2745623	9.498614
858	736164	631628712	29.2916370	9.502307
859	737881	633839779	29.3087018	9.505998
860	739600	636056000	29.3257566	9.509685
861	741321	638277381	29.3428015	9.513369
862	743044	640503928	29.3598365	9.517051
863	744769	642735647	29.3768616	9.520730
864	746496	644972544	29.3938769	9.524406
865	748225	647214625	29.4108823	9.528079
866	749956	649461896	29.4278779	9.531749
867	751689	651714363	29.4448637	9.535417
868	753424	653972032	29.4618397	9.539081
869	755161	656234909	29.4788059	9.542743
870	756900	658503000	29.4957624	9.546402
871	758641	660776311	29.5127091	9.550058
872	760384	663054848	29.5296461	9.553712
873	762129	665338617	29.5465734	9.557363
874	763876	667627624	29.5634910	9.561010
875	765625	669921875	29.5803989	9.564655
876	767376	672221376	29.5972972	9.568297
877	769129	674526133	29.6141858	9.571937
878	770884	676836152	29.6310648	9.575574
879	772641	679151439	29.6479325	9.579208
880	774400	681472000	29.6647939	9.582839
881	776161	683797841	29.6816442	9.586468
882	777924	686128968	29.6984848	9.590093
883	779689	688465387	29.7153159	9.593716
884	781456	690807104	29.7321375	9.597337
885	783225	693154125	29.7489496	9.600954
886	784996	695506456	29.7657521	9.604569
887	786769	697864103	29.7825452	9.608181
888	788544	700227072	29.7993289	9.611791
889	790321	702595369	29.8161030	9.615397
890	792100	704969000	29.8328678	9.619001
891	793881	707347971	29.8496231	9.622603

Numb.	Square.	Cube.	Square Root.	Cube Root.
892	795664	709732288	29.8663690	9.626201
893	797449	712121957	29.8831056	9.629797
894	799236	714516984	29.8998328	9.633390
895	801025	716917375	29.9165506	9.636981
896	802816	719323136	29.9332591	9.640569
897	804609	721734273	29.9499583	9.644154
898	806404	724150792	29.9666481	9.647736
899	808201	726572699	29.9833287	9.651316
900	810000	729000000	30.0000000	9.654893
901	811804	731432701	30.0166620	9.658468
902	813604	733870808	30.0333148	9.662040
903	815409	736314327	30.0499584	9.665609
904	817216	738763264	30.0665928	9.669176
905	819025	741217625	30.0832179	9.672740
906	810836	743677416	30.0998339	9.676301
907	822649	746142643	30.1164407	9.679860
908	824464	748613312	30.1330383	9.683416
909	826281	751089429	30.1496269	9.686970
910	828100	753571000	30.1632063	9.690521
911	829921	756058031	30.1827765	9.694069
912	831744	758550528	30.1993377	9.697615
913	833569	761048497	30.2158899	9.701158
914	835396	763551944	30.2324329	9.704698
915	837225	766060875	30.2489669	9.708236
916	839056	768575296	30.2654919	9.711772
917	840889	771095213	30.2820079	9.715305
918	842724	773620632	30.2985148	9.718835
919	844561	776151559	30.3150128	9.722363
920	846400	778688000	30.3315018	9.725888
921	848241	781229961	30.3479818	9.729410
922	850084	783777448	30.3644529	9.732930
923	851929	786330467	30.3809151	9.736448
924	853776	788889024	30.3973683	9.739963
925	855625	791453125	30.4138127	9.743475
926	857476	794022776	30.4302481	9.746985
927	859329	796597983	30.4466747	9.750493
928	861184	799178752	30.4630924	9.753998
929	863041	801765089	30.4795013	9.757500
930	864900	804357000	30.4959014	9.761000

Numb.	Square.	Cube.	Square Root.	Cube Root.
931	866761	806954491	30.5122926	9.764497
932	868624	809557568	30.5286750	9.767992
933	870489	812166237	30.5450487	9.771484
934	872356	814780504	30.5614136	9.774974
935	874225	817400375	30.5777697	9.778461
936	876096	820025856	30.5941171	9.782946
937	877969	822656953	30.6104557	9.785428
938	879844	825293672	30.6267857	9.788908
939	881721	827936019	30.6431069	9.792386
940	883600	830584000	30.6594194	9.795861
941	885481	833237621	30.6757233	9.799333
942	887364	835896888	30.6920185	9.802803
943	889249	838561807	30.7083051	9.806271
944	891136	341232384	30.7245830	9.809736
945	893025	843908625	30.7408523	9.813198
946	894916	846590536	30.7571130	9.816659
947	896809	849278123	30.7733651	9.820117
948	898704	851971392	30.7896086	9.823572
949	900601	854670349	30.8058436	9.827025
950	902500	857375000	30.8220700	9.830475
951	904401	860085351	30.8382879	9.833923
952	906304	862801408	30.8544972	9.837369
953	908209	865523177	30.8706981	9.840812
954	910116	868250664	30.8868904	9.844253
955	912025	870983875	30.9030743	9.847692
956	913936	873722816	30.9192497	9.851128
957	915849	876467493	30.9354166	9.854561
958	917764	879217912	30.9515751	9.857992
959	919681	881974079	30.9677251	9.861421
960	921600	884736000	30.9838668	9.864848
961	923521	887503681	31.0000000	9.868272
962	925444	890277128	31.0161248	9.871694
963	927369	893056347	31.0322413	9.875113
964	929296	895841344	31.0483494	9.878530
965	931225	898632125	31.0644491	9.881945
966	933156	901428696	31.0805405	9.885357
967	935089	904231063	31.0966236	9.888767
968	937024	907039232	31.1126984	9.892174
969	938961	909853209	31.1287648	9.895580

Numb.	Square.	Cube.	Square Root.	Cube Root.
970	940900	912673000	31.1448230	9.898983
971	942841	915498611	31.1608729	9.902383
972	944784	918330048	31.1769145	9.905781
973	946729	921167317	31.1929479	9.909177
974	948676	924010424	31.2089731	9.912571
975	950625	926859375	31.2249900	9.915962
976	952576	929714176	31.2409987	9.919351
977	954529	932574833	31.2569992	9.922738
978	956484	935441352	31.2729915	9.926122
979	958441	938313739	31.2889757	9.929504
980	960400	941192000	31.3049517	9.932883
981	962361	944076141	31.3209195	9.936261
982	964324	946966168	31.3368792	9.939636
983	966289	949862087	31.3528308	9.943009
984	968256	952763904	31.3687743	9.946379
985	970225	955671625	31.3847097	9.949747
986	972196	958585256	31.4006369	9.953113
987	974169	961504803	31.4165561	9.956477
988	976144	964430272	31.4324673	9.959839
989	978121	967361669	31.4483704	9.963198
990	980100	970299000	31.4642654	9.966554
991	982081	973242271	31.4801525	9.969909
992	984064	976191488	31.4960315	9.973262
993	986049	979146657	31.5119025	9.976612
994	988036	982107784	31.5277655	9.979959
995	990025	985074875	31.5436206	9.983304
996	992016	988047936	31.5594677	9.986648
997	994009	991026973	31.5753068	9.989990
998	996004	994011992	31.5911380	9.993328
999	998001	997002999	31.6069613	9.996665
1000	1000000	1000000000	31.6227766	10.000000

AREAS OF CIRCLES.

Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.
Inch	Inches.	Inch	Inches.	Inch	Inches.	Inch	Inches.	Inch	Inches.
1	.785	6	28.274	11	95,033	16	201.06	21	346.36
1/8	.994	1/8	29.464	1/8	97.205	1/8	204.21	1/8	350.49
1/4	1.227	1/4	30.679	1/4	99.402	1/4	207.39	1/4	354.65
3/8	1.484	3/8	31.919	3/8	101.62	3/8	210.59	3/8	358.84
1/2	1.767	1/2	33.183	1/2	103.86	1/2	213.82	1/2	363.05
5/8	2.073	5/8	34.471	5/8	106.13	5/8	217.07	5/8	367.28
3/4	2.405	3/4	35.784	3/4	108.43	3/4	220.35	3/4	371.54
7/8	2.761	7/8	37.122	7/8	110.75	7/8	223.65	7/8	375.82
2	3.141	7	38.484	12	113.09	17	226.98	22	380.13
1/8	3.546	1/8	39.871	1/8	115.46	1/8	230.33	1/8	384.46
1/4	3.976	1/4	41.282	1/4	117.85	1/4	233.70	1/4	388.82
3/8	4.430	3/8	42.718	3/8	120.27	3/8	237.10	3/8	393.20
1/2	4.908	1/2	44.178	1/2	122.71	1/2	240 52	1/2	397.60
5/8	5.411	5/8	45.663	5/8	125.18	5/8	243.97	5/8	402.03
3/4	5.939	3/4	47.173	3/4	127.67	3/4	247.45	3/4	406.49
7/8	6.491	7/8	48.707	7/8	130.19	7/8	250.94	7/8	410.97
3	7.068	8	50.265	13	132.73	18	254.46	23	415.47
1/8	7.669	1/8	51.848	1/8	135.29	1/8	258.01	1/8	420.00
1/4	8 295	1/4	53.456	1/4	137.88	1/4	261.58	1/4	424.55
3/8	8.946	3/8	55.088	3/8	140.50	3/8	265.18	3/8	429.13
1/2	9.621	1/2	56.745	1/2	143.13	1/2	268.80	1/2	433.73
5/8	10.320	5/8	58.426	5/8	145.80	5/8	272.44	5/8	438.36
3/4	11.044	3/4	60 132	3/4	148.48	3/4	276.11	3/4	443.01
7/8	11.793	7/8	61.862	7/8	151.20	7/8	279.81	7/8	447.69
4	12.566	9	63.617	14	153.93	19	283.52	24	452.39
1/8	13.364	1/8	65.396	1/8	156.69	1/8	287.27	1/8	457.11
1/4	14.186	1/4	67.200	1/4	159.48	1/4	291.03	1/4	461.80
3/8	15.033	3/8	69.029	3/8	162.29	3/8	294.83	3/8	466.63
1/2	15.904	1/2	70.882	1/2	165.13	1/2	298·64	1/2	471.43
5/8	16.800	5/8	72.759	5/8	167.98	5/8	302.48	5/8	476.25
3/4	17.720	3/4	74.662	3/4	170.87	3/4	306.35	3/4	481.10
7/8	18.665	7/8	76.588	7/8	173.78	7/8	310.24	7/8	485.97
5	19.635	10	78.540	15	176.71	20	314.16	25	490.87
1/8	20.629	1/8	80.515	1/8	179.67	1/8	318.09	1/8	495.79
1/4	21.647	1/4	82.516	1/4	182.65	1/4	322.06	1/4	500.74
3/8	22 690	3/8	84.540	3/8	185.66	3/8	326.05	3/8	505.71
1/2	23.758	1/2	86.590	1/2	188.69	1/2	330.06	1/2	510.70
5/8	24.850	5/8	88.664	5/8	191.74	5/8	334.10	5/8	515.72
3/4	25.967	3/4	90.762	3/4	194.82	3/4	338.16	3/4	520.76
7/8	27.108	7/8	92.885	7/8	197.93	7/8	342.25	7/8	525.83

Dia. Inch	Area. Inches.	Dia. Inch	Area. Inches.	Dia. Inch	Area. Inches.	Dia. Inch	Area. Inches.	Dia. Inch	Area Inches.
26	530.93	32	804.24	38	1134.1	44	1520.5	50	1963.5
1/8	536.04	1/8	810.54	1/8	1141.5	1/8	1529.1	1/8	1973.3
1/4	541.18	1/4	816.86	1/4	1149.0	1/4	1537.8	1/4	1983.1
3/8	546.35	3/8	823.21	3/8	1156.6	3/8	1546.5	3/8	1993.0
1/2	551.54	1/2	829.57	1/2	1164.1	1/2	1555.2	1/2	2002.9
5/8	556.76	5/8	835.97	5/8	1171.7	5/8	1564.0	5/8	2012.8
3/4	562.00	3/4	842.39	3/4	1179.3	3/4	1572.8	3/4	2022.8
7/8	567.26	7/8	848.83	7/8	1186.9	7/8	1581.6	7/8	2032.8
27	572.55	33	855.30	39	1194.5	45	1590.4	51	2042.8
1/8	577.87	1/8	861 79	1/8	1202.2	1/8	1599.2	1/8	2052.8
1/4	583.20	1/4	868.30	1/4	1209.9	1/4	1608.1	1/4	2062.9
3/8	588.57	3/8	874.84	3/8	1217.6	3/8	1617.0	3/8	2072.9
1/2	593.95	1/2	881.41	1/2	1225.4	1/2	1625.9	1/2	2083.0
5/8	599.37	5/8	888.00	5/8	1233.1	5/8	1634.9	5/8	2093.2
3/4	604.80	3/4	894.61	3/4	1240.9	3/4	1643.8	3/4	2103.3
7/8	610.26	7/8	901.25	7/8	1248.7	7/8	1652.8	7/8	2113.5
28	615.75	34	907.92	40	1256.6	46	1661.9	52	2123.7
1/8	621.26	1/8	914.61	1/8	1264.5	1/8	1670.9	1/8	2133.9
1/4	626.79	1/4	921.32	1/4	1272.3	1/4	1680.0	1/4	2144.1
3/8	632.35	3/8	928.06	3/8	1280.3	3/8	1689.1	3/8	2154.4
1/2	637.94	1/2	934.82	1/2	1288.2	1/2	1698.2	1/2	2164.7
5/8	643.54	5/8	941.60	5/8	1296.2	5/8	1707.3	5/8	2175.0
3/4	649.18	3/4	948.41	3/4	1304.2	3/4	1716.5	3/4	2185.4
7/8	654.83	7/8	955.25	7/8	1312.2	7/8	1725.7	7/8	2195.7
29	660.52	35	962.11	41	1320.2	47	1734.9	53	2206.1
1/8	666.22	1/8	968.99	1/8	1328.3	1/8	1744.1	1/8	2216.6
1/4	671.95	1/4	975.90	1/4	1336.4	1/4	1753.4	1/4	2227.0
3/8	677.71	3/8	982.84	3/8	1344.5	3/8	1762.7	3/8	2237.5
1/2	683.49	1/2	989.80	1/2	1352.6	1/2	1772.0	1/2	2248.0
5/8	689.29	5/8	996.78	5/8	1360.8	5/8	1781.3	5/8	2258.5
3/4	695.12	3/4	1003.7	3/4	1369.0	3/4	1790.7	3/4	2269.0
7/8	700.98	7/8	1010.8	7/8	1377.2	7/8	1800.1	7/8	2279.6
30	706.86	36	1017.8	42	1385.4	48	1809.5	54	2290.2
1/8	712.76	1/8	1024.9	1/8	1393.7	1/8	1818.9	1/8	2300.8
1/4	718.69	1/4	1032.0	1/4	1401.9	1/4	1828.4	1/4	2311.4
3/8	724.64	3/8	1039.1	3/8	1410.2	3/8	1837.9	3/8	2322.1
1/2	730.61	1/2	1046.3	1/2	1418.6	1/2	1847.4	1/2	2332.8
5/8	736.61	5/8	1053.5	5/8	1426.9	5/8	1858.9	5/8	2343.5
3/4	742.64	3/4	1060.7	3/4	1435.3	3/4	1866.5	3/4	2354.2
7/8	748.69	7/8	1067.9	7/8	1443.7	7/8	1876.1	7/8	2365.0
31	754.76	37	1075.2	43	1452.2	49	1885.7	55	2375.8
1/8	760.86	1/8	1082.4	1/8	1460.6	1/8	1895.3	1/8	2386.6
1/4	766.99	1/4	1089.7	1/4	1469.1	1/4	1905.0	1/4	2397.4
3/8	773.14	3/8	1097.1	3/8	1477.6	3/8	1914.7	3/8	2408.3
1/2	779.31	1/2	1104.4	1/2	1486.1	1/2	1924.4	1/2	2419.2
5/8	785.51	5/8	1111.8	5/8	1494.7	5/8	1934.1	5/8	2430.1
3/4	791.73	3/4	1119.2	3/4	1503.3	3/4	1943.9	3/4	2441.0
7/8	797.97	7/8	1126.6	7/8	1511.9	7/8	1953.6	7/8	2452 0

Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.
Inch	Inches.	Inch	Inches.	Inch	Inches.	Inch	Inches.	Inch	Inches.
56	2463.0	62	3019.0	68	3631.6	74	4300.8	80	5026.5
⅛	2474.0	⅛	3031.2	⅛	3645.0	⅛	4315.3	⅛	5042.2
¼	2485.0	¼	3043.4	¼	3658.4	¼	4329.9	¼	5058.0
⅜	2496.1	⅜	3055.7	⅜	3671.8	⅜	4344.5	⅜	5073.7
½	2507.1	½	3067.9	½	3685.2	½	4359.1	½	5089.5
⅝	2518.2	⅝	3080.2	⅝	3698.7	⅝	4373.8	⅝	5105.4
¾	2529.4	¾	3092.5	¾	3712.2	¾	4388.4	¾	5121.2
⅞	2540.5	⅞	3104.8	⅞	3725.7	⅞	4403.1	⅞	5137.1
57	2551.7	63	3117.2	69	3739.2	75	4417.8	81	5153.0
⅛	2562.9	⅛	3129.6	⅛	3752.8	⅛	4432.6	⅛	5168.9
¼	2574.1	¼	3142.0	¼	3766.4	¼	4447.3	¼	5184.8
⅜	2585.4	⅜	3154.4	⅜	3780.0	⅜	4462.1	⅜	5200.8
½	2596.7	½	3166.9	½	3793.6	½	4476.9	½	5216.8
⅝	2608.0	⅝	3179.4	⅝	3807.3	⅝	4491.8	⅝	5232.8
¾	2619.3	¾	3191.9	¾	3821.0	¾	4506.6	¾	5248.8
⅞	2630.7	⅞	3204.4	⅞	3834.7	⅞	4521.5	⅞	5264.9
58	2642.0	64	3216.9	70	3848.4	76	4536.4	82	5281.0
⅛	2653.4	⅛	3229.5	⅛	3862.2	⅛	4551.4	⅛	5297.1
¼	2664.9	¼	3242.1	¼	3875.9	¼	4566.3	¼	5313.2
⅜	2676.3	⅜	3254.8	⅜	3889.8	⅜	4581.3	⅜	5329.4
½	2687.8	½	3267.4	½	3903.6	½	4596.3	½	5345.6
⅝	2699.3	⅝	3280.1	⅝	3917.4	⅝	4611.3	⅝	5361.8
¾	2710.8	¾	3292.8	¾	3931.3	¾	4626.4	¾	5378.0
⅞	2722.4	⅞	3305.5	⅞	3945.2	⅞	4641.5	⅞	5394.3
59	2733.9	65	3318.3	71	3959.2	77	4656.6	83	5410.6
⅛	2745.5	⅛	3331.0	⅛	3973.1	⅛	4671.7	⅛	5426.9
¼	2757.1	¼	3343.8	¼	3987.1	¼	4686.9	¼	5443.2
⅜	2768.8	⅜	3356.7	⅜	4001.1	⅜	4702.1	⅜	5459.6
½	2780.5	½	3369.5	½	4015.1	½	4717.3	½	5476.0
⅝	2792.2	⅝	3382.4	⅝	4029.2	⅝	4732.5	⅝	5492.4
¾	2803.9	¾	3395.3	¾	4043.2	¾	4747.7	¾	5508.8
⅞	2815.6	⅞	3408.2	⅞	4067.3	⅞	4763.0	⅞	5525.3
60	2827.4	66	3421.2	72	4071.5	78	4778.3	84	5541.7
⅛	2839.2	⅛	3434.1	⅛	4085.6	⅛	4793.7	⅛	5558.2
¼	2851.0	¼	3447.1	¼	4099.8	¼	4809.0	¼	5574.8
⅜	2862.8	⅜	3460.1	⅜	4114.0	⅜	4824.4	⅜	5591.3
½	2874.7	½	3473.2	½	4128.2	½	4839.8	½	5607.9
⅝	2886.6	⅝	3486.3	⅝	4142.5	⅝	4855.2	⅝	5624.5
¾	2898.5	¾	3499.3	¾	4156.7	¾	4870.7	¾	5641.1
⅞	2910.5	⅞	3512.5	⅞	4171.0	⅞	4886.1	⅞	5657.8
61	2922.4	67	3525.6	73	4185.3	79	4901.6	85	5674.5
⅛	2934.4	⅛	3538.8	⅛	4199.7	⅛	4917.2	⅛	5691.2
¼	2946.4	¼	3552.0	¼	4214.1	¼	4932.7	¼	5707.9
⅜	2958.5	⅜	3565.2	⅜	4228.5	⅜	4948.3	⅜	5724.6
½	2970.5	½	3578.4	½	4242.9	½	4963.9	½	5741.4
⅝	2982.6	⅝	3591.7	⅝	4257.3	⅝	4979.5	⅝	5758.2
¾	2994.7	¾	3605.0	¾	4271.8	¾	4995.1	¾	5775.0
⅞	3006.9	⅞	3618.3	⅞	4286.3	⅞	5010.8	⅞	5791.9

Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.	Dia.	Area.
Inch	Inches.	Inch	Inches.	Inch	Inches.	Inch	Inches.	Inch	Inches.
86	5808.8	89	6221.1	92	6647.6	95	7088.2	98	7542.9
1/8	5825.7	1/8	6238.6	1/8	6665.7	1/8	7106.9	1/8	7562.2
1/4	5842.6	1/4	6256.1	1/4	6683.8	1/4	7125.5	1/4	7581.5
3/8	5859.5	3/8	6273.6	3/8	6701.9	3/8	7144.3	3/8	7600.8
1/2	5876.5	1/2	6291.2	1/2	6720.0	1/2	7163.0	1/2	7620.1
5/8	5893.5	5/8	6308.8	5/8	6738.2	5/8	7181.8	5/8	7639.4
3/4	5910.5	3/4	6326.4	3/4	6756.4	3/4	7200.5	3/4	7658.8
7/8	5927.6	7/8	6344.0	7/8	6776.4	7/8	7219.4	7/8	7678.2
87	5944.6	90	6361.7	93	6792.9	96	7238.2	99	7697.7
1/8	5961.7	1/8	6379.4	1/8	6811.1	1/8	7257.1	1/8	7717.1
1/4	5978.9	1/4	6397.1	1/4	6829.4	1/4	7275.9	1/4	7736.6
3/8	5996.0	3/8	6414.8	3/8	6847.8	3/8	7294.9	3/8	7756.1
1/2	6013.2	1/2	6432.6	1/2	6866.1	1/2	7313.8	1/2	7775.6
5/8	6030.4	5/8	6450.4	5/8	6884.5	5/8	7332.8	5/8	7795.2
3/4	6047.6	3/4	6468.2	3/4	6902.9	3/4	7351.7	3/4	7814.7
7/8	6064.8	7/8	6486.0	7/8	6921.3	7/8	7370.7	7/8	7834.3
88	6082.1	91	6503.8	94	6939.7	97	7389.8	100	7854.0
1/8	6099.4	1/8	6521.7	1/8	6958.2	1/8	7408.8		
1/4	6116.7	1/4	6539.6	1/4	6976.7	1/4	7427.9		
3/8	6134.0	3/8	6557.6	3/8	6995.2	3/8	7447.0		
1/2	6151.4	1/2	6575.5	1/2	7013.8	1/2	7466.2		
5/8	6168.8	5/8	6593.5	5/8	7032.3	5/8	7485.3		
3/4	6186.2	3/4	6611.5	3/4	7050.9	3/4	7504.5		
7/8	6203.6	7/8	6629.5	7/8	7069.5	7/8	7523.7		

USE OF THIS TABLE.

To find, by inspection, the area of any circle from 1 to 100 inches, of which the diameter is given.

MISCELLANIES.

Required the weight of a cast iron ball of 3 inches diameter, supposing the weight of a cubic inch of the metal to be 0.258 libs avoirdupois? HUTTON.

$3^3 \times .5236 \times .258 = 3.6473976$ libs avoir.

Required the weight of a hollow spherical iron shell, 5 inches in diameter, the thickness of the metal being 1 inch? HUTTON.

$5^3 \times .5236 \times .258 = 16.88610 - 3.6473976 = 13.23871$ libs avoirdupois.

It is proposed to determine the proportional quantities of matter in the earth and moon; the density of the former being to that of the latter, as 10 to 7, and their diameters, as 7930 to 2160. HUTTON.

$\frac{7930^3 \times 10}{2160^3 \times 7} = 71$ nearly: that is, 71 to 1 nearly.

There are two bodies, of which the one contains 25 times the matter of the other, or is 25 times heavier; but the less moves with 1000 times the velocity

of the greater; in what proportion then are the momenta, or forces with which they move? HUTTON.

$\frac{1000}{25} = 40$; that is, the less moves with a force 40 times greater.

It is proposed to divide the Beam of a Steel-yard, or to find the points of division where the weights of 1 2 3 4, &c. libs on the one side, will just balance a constant weight of 95 libs; at the distance of 2 inches on the other side of the fulcrum, the weight of the Beam being 10 libs, and its whole length 36 inches.

$36 : 10 :: 2 : 55 = \frac{20.000}{36} = .55$ weight of short arm.

$2 =$ length of short arm.

$36 - 2 = 34 =$ length of long arm.

$10 - .55 = 9.45 =$ weight of long arm.

$95 \times 2 = 190$ momentum of weight at end of short arm.

$.55 \times 1 = .55$ do. of short arm.

190.55 whole momentum of weight and arm.

$\frac{9.45 \times 34}{2} = 160.65$ momentum of long arm.

$190.55 - 160.65 = 29.90$, or 30, the excess of momentum.

$\frac{30}{1} = 30$ inches from the fulcrum for the one lib weight.

$\frac{30}{2} = 15$ . . . . . . two do.

$\frac{30}{3} = 10$ . . . . . . three do.

$\frac{30}{4} = 7\frac{1}{2}$ . . . . . . four do.

How long after firing the tower guns, may the report be heard at Shooter's-Hill, supposing the distance to be 8 miles in a straight line?

$$8 \times \frac{14}{3} = 8 \times 14 = \frac{112}{3} = 37\tfrac{1}{3} \text{ seconds.}$$

If a lever, 40 effective inches long, will, by a certain power thrown successively on it, in 13 hours, raise a weight 104 feet; in what time will two other levers, each 18 effective inches long, raise an equal weight 73 feet.

If 40 : 104 : : 18 : 46.8, one of the 18 inch levers would raise in 13 hours.

If 46.8 × 2 = 93.6 : 13 : : 73 : 10 hours 8⅓ minutes, time to raise the weight 73 feet.

There are two bodies, the one of which weighs 100 libs, the other 60 libs; but the lightest is impelled by a force eight times greater than the other; the proportion of the velocities with which these bodies move, is required? HUTTON.

$\frac{100 \times 8 \times 3}{60} = 40$, that is, the velocity of the greater is to that of the less, as 3 to 40.

Supposing one body to move 30 times swifter than another; as also the swifter to move 12 minutes, the other only 1; what differenee will there be

between the spaces described by them, supposing the last has removed 5 feet?

$30 \times 5 = 150$ feet in first minute. $150 \times 12 = 1800 - 5 = 1795$ feet difference between the spaces described.

If a Stone be $19\frac{1}{2}$ seconds in descending from the top of a precipice to the bottom, what is its height?

$19.5^2 = 380.25 \times 16\frac{1}{12} = 6115.6875$ feet.

There are two bodies, the one of which has passed over 50 miles, the other only 5; and the first had moved with 5 times the celerity of the second; what is the ratio of the times they have been in describing those spaces?

25 velocity of first. $\frac{\text{Space } 5}{\text{Velocity } 5} = 1$ time of the second.
5 do. of second.

$\frac{\text{Space } 50}{\text{Velocity } 25} = 2$ time of the second;

therefore the ratio of the times will be 2 to 1.

Counted 17″ between the time of seeing the flash, and hearing the report of a gun, what is the distance from it?

Sound flies through the air uniformly at the rate of about 1142 feet in 1 second of time, or a mile in $4\frac{2}{3}$ or $\frac{14}{3}$ seconds.

$$\frac{1142 \times 17}{5280} = \overset{\text{M. Feet.}}{3.3574}, \text{ or } 3\tfrac{2}{3} \text{ miles distance.}$$

How far off was the cloud from which thunder issued, whose report was 5 pulsations in the wrist, after the flash of lightning, counting 75 pulsations to a minute?

$$\frac{60 \times 5}{75} = 4 \times 1142 = 4568 \text{ feet, or } 1522\tfrac{2}{3} \text{ yards.}$$

A Stone being let fall into a well, it was observed, that, after being dropped, it was 10 seconds before the sound of the fall at the bottom reached the ear; what is the depth of the well?

Let x be the depth in feet,
t the time of descent.
therefore $10-t$ will be the time of the sound's ascent.

Then $16.083\,t^2 = x$
$\overline{10-t} \times 1142 = x$ consequently
$\frac{16.083\,t^2 = 11420-1142\,t}{16.083} = t^2+71\,t = 710$ nearly
complete the square, $t^2 + 71t + 1260.25 = 1970.25$
and $t + 35.5 = \sqrt{1970.25} = 44.38$, so that
$t = 44.38-35.5 = 8.88$ seconds, and $x = \overline{8.88}^2 \times 16.083 = 1267.34$ feet nearly.

Suppose in a circle of 3 miles circumference, two horses are started at the same instant, from the same point, A runs at the velocity of 7 miles an hour, B runs at the velocity of 5 miles an hour; when will A overtake B?

3 miles circumference = 15840 feet.
A runs at 616 feet per minute.
B runs at 440 do. do.
$\overline{176}$ = difference of speed.

$\frac{15840}{176} = 90$ minutes—*i. e.*

A will overtake B in 90 minutes, running $616 \times 90 = 55440$ feet $=3$ rounds of the circle, and 7920 feet.

A left town 50 minutes before B, B being informed of it, followed at the rate of 6 miles an hour, and made up to A 9 miles from town; at what rate did A travel?

B's rate is 528 feet per minute.

$$5280 \times 9 = \frac{47520}{528} = 90'$$

B made up to A in 90′ $\frac{90' \times 528}{90' + 50} = 339\frac{6}{14}$ feet, the speed at which A travelled per minute.

PROBLEM.

To determine how far a man, who pushes with the force of 100 *libs, can thrust a sponge into a piece of ordnance, whose diameter is* 5 *inches, and length* 10 *feet, when the barometer stands at* 30 *inches: the vent or touch-hole being stopped, and the sponge having no windage, that is, fitting the bore quite close?*

A column of quicksilver 30 inches high, and 5 inches diameter, is $5^2 \times 30 \times .7854 = 589.05$ inches;

which, at 8.102 oz. each inch, weighs 4772.48 oz. or 298.28 libs, which is the pressure of the atmosphere alone, being equal to the elasticity of the air in its natural state; to this adding the 100 libs, gives 398.28 libs, the whole external pressure. Then, as the spaces which a quantity of air possesses under different pressures, are in the reciprocal ratio of those pressures, it will be, as 398.28 : 298.28 : : 10 feet or 120 inches : 90 inches nearly, the space occupied by the air; therefore 120 — 90 = 30 inches, is the distance sought.

From this Problem of Dr. Hutton's the following formula is easily obtained, *viz.*

a = whole length of tube, or volume of air in its natural state.
b = pressure of air in its natural state.
d = space occupied by the compressed air.
n = distance the air is compressed.
x = force exerted to compress the air, or the pressure of the air when compressed.

$$b + x : b :: a : d$$

$$a = \frac{(b+x)\,d}{b} \qquad d = \frac{a\,b}{b+x}$$

$$b = \frac{x\,d}{a-d} \qquad x = \frac{a\,b}{d} - b$$

$$n = a - d$$

The following Table shows the force of air, when compressed at — times its volume, calculated according to the preceding Theorem.

Times the volume.	Pressure on the square inch.		Times the volume.	Pressure on the square inch.		Times the volume.	Pressure on the square inch.	
	Libs.	*Oz.*		*Libs.*	*Oz.*		*Libs.*	*Oz.*
2	15	3	18	258	3	34	501	3
4	45	9	20	288	9	36	531	9
6	75	15	22	318	15	38	561	15
8	106	5	24	349	5	40	592	5
10	136	11	26	379	11	42	622	11
12	167	1	28	410	1	44	653	1
14	197	7	30	440	7	46	683	7
16	227	13	32	470	13	48	713	13

To find the pressure of air at any number of volumes, is very easy and simple—Multiply the pressure of the air, say 15 libs on the square inch, by the number of volumes required, then deduct one volume for the circumambient atmosphere, or multiply 15 libs by one less than the number of volumes or atmospheres required.

TABLE.

OF THE PROPERTIES OF VARIOUS BODIES.

BODIES.	Specific Gravity.	Weight of a cubic foot.	Will bear on square inch without permanent Alteration.	Melts at degrees.	Cohesive force of a square inch.	Crushed by a force on square inch.	Absorbs of its weight of water.	Strength compared with cast iron.
WOODS.		*Libs.*	*Libs.*					
Ash	0.75	47.5	3540	- - -	- - -	- - -	- - -	.23
Beech	0.696	45.3	2360	- - -	- - -	- - -	- - -	.15
Elm	0.544	34.	3240	- - -	- - -	- - -	- - -	.21
Yellow & Red Fir	0.557	34.8	4290	- - -	- - -	- - -	- - -	.3
White do.	0.47	29.3	3630	- - -	- - -	- - -	- - -	.23
Mahogany	0.56	35.	3800	- - -	- - -	- - -	- - -	.24
English Oak	0.83	52.	3960	- - -	- - -	- - -	- - -	.25
Ame. Yellow Pine	0.46	26.75	3900	- - -	- - -	- - -	- - -	.25
Larch	0.560	35.	2065	- - -	- - -	- - -	- - -	.136
METALS.								
Cast Brass . .	8.37	506.25	6700	1869°	18000	- - -	- - -	.435
Cast Iron . . .	7.207	450.	15300	3479°	- - -	93000	- - -	1.
Copper .	8.75	549.	- - -	2548°	33000			
Malleable Iron . .	7.6	475.	17800	- - -	- - -	- - -	- - -	1.12
Hammer'd do.	. . . .	487.						
Cast Lead . .	11.352	709.5	1500	612°	- - -	- - -	- - -	.096
Steel . .	7.84	490.	- - -	- - -	130000			
Cast Tin . . .	7.291	455.7	2880	442°	- - -	- - -	- - -	.182
Cast Zinc . .	7.028	439.25	5700	648°	- - -	- - -	- - -	.365
Cast Gun Metal	8.153	509.13	10000	- - -	- - -	- - -	- - -	.65
STONE, &c.								
Brick . .	1.841	115.	- - -	- - -	275	562	.066	
Chalk .	2.315	144.7	- - -	- - -	- - -	500		
Clay . . .	2.	125.						
Aberdeen Granite	2.625	164.	- - -	- - -	- - -	10910		
White Marble	2.706	169.	- - -	- - -	1811	6060		
Red Porphyry	2.871	179.	- - -	- - -	- - -	35568		
Welsh Slate	2.752	172.	- - -	- - -	11500			
Portland Stone	2.113	132.	- - -	- - -	857	3729	.0625	
Bath do.	1.975	123.4	- - -	- - -	478	- - -	.077	
Craigleith do.	2.362	147.6	- - -	- - -	772	5490	.0158	
Dundee do.	2.621	163.8	- - -	- - -	2661	6630	.002	

By the last column of this Table, the Rules for the strength of cast iron can be applied to the various bodies.

TABLE OF THE WEIGHT OF CAST IRON PIPES.

Bore.	Thick.	Long.	Weight.			Bore.	Thick.	Long.	Weight.			Bore.	Thick.	Long.	Weight.		
1	¼	3ft6	0	0	12	6½	⅝	9	3	2	21	11½	¾	9	7	2	8
	⅜	3ft6	0	0	21		¾	9	4	1	21		1	9	10	1	2
1½	¼	4ft6	0	0	21		1	9	6	0	14	12	½	9	5	0	24
	⅜	4ft6	0	1	4	7	⅜	9	2	1	7		⅝	9	6	2	8
2	¼	6	0	1	8		½	9	3	0	7		¾	9	7	3	20
	⅜	6	0	2	0		⅝	9	3	3	20		1	9	10	3	0
2½	¼	6	0	1	16		¾	9	4	3	5	12½	½	9	5	1	16
	⅜	6	0	2	10		1	9	6	2	4		⅝	9	6	3	9
	½	6	0	3	10	7½	⅜	9	2	2	4		¾	9	8	1	0
3	¼	9	0	2	20		½	9	3	1	6		1	9	11	0	21
	⅜	9	1	0	6		⅝	9	4	0	22	13	½	9	5	2	20
	½	9	1	1	12		¾	9	5	0	10		⅝	9	7	0	14
	⅝	9	1	3	6		1	9	7	0	0		¾	9	8	2	7
	¾	9	2	1	0	8	½	9	3	2	4		1	9	11	2	12
3½	¼	9	0	3	0		⅝	9	4	1	25	13½	½	9	5	3	7
	⅜	9	1	0	21		¾	9	5	1	18		⅝	9	7	1	12
	½	9	1	2	14		1	9	7	1	16		¾	9	8	3	16
	⅝	9	2	0	8	8½	½	9	3	3	2		1	9	11	3	24
	¾	9	2	2	0		⅝	9	4	2	26	14	½	9	6	0	4
4	⅜	9	1	1	10		¾	9	5	2	22		⅝	9	7	2	16
	½	9	1	3	12		1	9	7	3	8		¾	9	9	1	0
	⅝	9	2	1	12	9	½	9	4	0	0		1	9	12	1	14
	¾	9	2	3	21		⅝	9	5	0	4	14½	½	9	6	0	24
4½	⅜	9	1	2	2		¾	9	6	0	2		⅝	9	7	3	14
	½	9	2	0	4		1	9	8	0	26		¾	9	9	2	2
	⅝	9	2	2	14	9½	½	9	4	0	18		1	9	12	3	6
	¾	9	3	0	21		⅝	9	5	1	0	15	½	9	6	1	21
5	⅜	9	1	2	22		¾	9	6	1	6		⅝	9	8	0	14
	½	9	2	1	10		1	9	8	2	20		¾	9	9	3	7
	⅝	9	2	3	17	10	½	9	4	1	10		1	9	13	0	26
	¾	9	3	1	24		⅝	9	5	1	26		1¼	9	16	3	5
5½	⅜	9	1	3	10		¾	9	6	2	14	15½	½	9	6	2	14
	½	9	2	2	0		1	9	9	0	8		⅝	9	8	1	14
	⅝	9	3	0	18	10½	½	9	4	2	14		¾	9	10	0	10
	¾	9	3	3	7		⅝	9	5	3	7		1	9	13	2	17
	1	9	5	0	12		¾	9	7	0	0		1¼	9	17	1	6
6	⅜	9	2	0	0		1	9	9	2	0	16	½	9	7	0	22
	½	9	2	2	21	11	½	9	4	3	14		⅝	9	8	3	7
	⅝	9	3	1	17		⅝	9	6	0	11		¾	9	10	1	20
	¾	9	4	0	16		¾	9	7	1	7		1	9	14	0	8
	1	9	5	2	20		1	9	9	3	20		1¼	9	17	3	14
6½	⅜	9	2	0	16	11½	½	9	5	0	7		1½	9	21	3	4
	½	9	2	3	20		⅝	9	6	1	12		2	9	29	3	21

The foregoing Table of the weight of cast iron pipes, gives the length of pipe according to the diameter of bore, as generally used in practice.

Diameter of bore in inches.
Thickness of metal in inches.
Length of pipe in feet.

It is found to be of great use in making out estimates of pipes:—for instance, it is required to know the weight of a range of pipes 225 feet long, 7½ inches diameter of bore, and metal $\frac{5}{8}$ths of an inch thick.

9)225

25 pipes in the whole length.

One pipe weighs 4.0.22, which multiplied by 25, is equal to 104.3.18, or 5 tons, 4 cwt. 3 quarters, 18 libs, weight of the whole range.

The following is a Table of the velocity of motion, for boring cast iron cylinders, pumps, &c. and heavy turning, with fixed cutters.

It will be observed that the surface bored is constantly the same, 78.54 feet per minute; this velocity is found to be the most advantageous: a velocity greater than this, not only takes the temper out of the cutters, but also causing more heat, expands the metal; and if the machine stops but for a few seconds, a mark is left from the contraction of the metal.

Turning has a velocity double to that of boring.

TABLE.

BORING.		TURNING.	
Inches diameter.	Revolutions of Bar per Minute.	Inches diameter.	Revolutions of Shaft per Minute.
1	25.	1	50.
2	12.5	2	25.
3	8.33	3	16.67
4	6.25	4	12.50
5	5.	5	10.
6	4.16	6	8.32
7	3.57	7	7.15
8	3.125	8	6.25
9	2.77	9	5.55
10	2.5	10	5.
15	1.66	15	3.33
20	1.25	20	2.50
25	1.	25	2.
30	0.833	30	1.667
35	0.714	35	1.430
40	0.625	40	1.250
45	0.56	45	1.12
50	0.5	50	1.
60	0.417	60	0.834
70	0.358	70	0.716
80	0.313	80	0.626
90	0.278	90	0.556
100	0.25	100	0.50

N. B. The progression of the cutters may be 1-16th of an inch for the first cut, and for the last 1-24th.

If hand-tools are employed in turning, the velocity may be considerably increased.

PROFESSOR FARISH'S ISOMETRICAL PERSPECTIVE.

In the course of Lectures which I deliver in the University of Cambridge, I exhibit models of almost all the more important machines which are in use in the manufactures of Britain.

The number of these is so large, that had each of them been permanent and separate, on a scale requisite to make them work, and to explain them to my audience, I should, independently of other objections, have found it difficult to have procured a warehouse large enough to contain them. I procured therefore an apparatus, consisting of what may be called a system of the first principles of machinery; that is, the separate parts, of which machines consist. These are made chiefly of metal, so strong, that they may be sufficient to perform even heavy work: and so adapted to each other, that they may be put together at pleasure, in every form, which the particular occasion requires.

Those parts are various: such as, loose brass wheels, the teeth of which all fit into one another: axes, of various lengths, on any part of which the wheel required may be fixed: bars, clamps, and frames; and whatever else might be necessary to build up the particular machines which are wanted

for one lecture. These models may be taken down, and the parts built up again in a different form for the lecture of the following day. As these machines, thus constructed for a temporary purpose, have no permanent existence in themselves, it became necessary to make an accurate representation of them on paper, by which my assistants might know how to put them together without the necessity of my continual superintendance. This might have been done, by giving three orthographic plans of each; one on the horizontal plane, and two on vertical planes at right angles to each other. But such a method, though in some degree in use among artists, would be liable to great objections. It would be unintelligible to an inexperienced eye; and even to an artist, it shows but very imperfectly that which is most essential, the connexion of the different parts of the engine with one another; though it has the advantage of exhibiting the lines parallel to the planes, on which the orthographic projections are taken on a perfect scale.

This will be easily understood, by supposing a cube to be the object represented. The ground plan would be a square representing both the upper and lower surfaces. And the two elevations would also be squares on two vertical planes, parallel to the other sides of the cube. The artist would have exhibited to him three squares; and he would have to discover how to put them together in the form of a cube, from the circumstance of there being two elevations and a ground plan. This method, therefore,

giving so little assistance on so essential a point, I thought unsatisfactory.

The taking a picture on the principles of common perspective, was the next expedient that suggested itself. And this might be adapted to the exhibition of a model, by taking a kind of bird's-eye view of the object, and having the plane of the picture, not as is most common in a drawing, perpendicular to the horizon, but to a line, drawn from the eye, to some principal part of the object. For example: in taking the picture of a cube, the eye might be placed in a distant point on the line which is formed by producing the diagonal of the cube. But to this common perspective, there are great objections. The lines, which in the cube itself are all equal, in the representation are unequal. So that it exhibits nothing like a scale. And to compute the proportions of the original from the representation, would be exceedingly difficult, and, for any useful purpose, impracticable: there is equal difficulty too, in computing the angles which represent the right angles of the cube. Neither does the representation appear correct, unless the eye of the person, who looks at it, be placed exactly in the point of sight. It is true that, as we are continually in the habit of looking at such perspective drawings, we get the habit of correcting, or rather overlooking the apparent errors which arise from the eye being out of the point of sight, and are therefore not struck with the appearance of incorrectness, which if we were unaccustomed to it, we should feel at once.

The kind of perspective which is the subject of this paper, though liable in a slight degree to the last-mentioned inconvenience, till the eye becomes used to it, I found much better adapted to the exhibition of machinery; I therefore determined to adopt it, and set myself to investigate its principles, and to consider how it might most easily be brought into practice.

It is preferable to the common perspective on many accounts, for such purposes. It is much easier and simpler in its principles. It is also, by the help of a common drawing-table, and two rulers,*

* It is unnecessary to describe the drawing-table any further than by observing that it ought to be so contrived, as to keep the paper steady on which the drawing is to be made.

Here should be a ruler in the form of the letter T to slide on one side of the drawing-table. The ruler should be kept, by small prominences on the under side, from being in immediate contact with the paper, to prevent its blotting the fresh drawn lines as it slides over them. And a second ruler, by means of a groove near one end on its under side, should be made to slide on the first. The groove should be wider than the breadth of the first ruler, and so fitted, that the second may at pleasure be put into either of the two positions represented in the plate, fig. 1, so as to contain with the former ruler, in either position, an angle of 60 degrees. The groove should be of such a size, that when its shoulders *a* and *d* are in contact with, and rest against the edges of the first ruler, the edge of the second ruler should coincide with *d e*, the side of an equilateral triangle described on *d g*, a portion of the edge of the first ruler; and when the shoulders *b* and *c* rest against the edges of the first ruler, the edge of the second should lie along *g e*, the other side of the equilateral triangle. The second ruler should have a little foot at *k* for the same purpose as the prominences on the first ruler, and both of them should have their edges divided into inches, and tenths, or eighths of inches.

It would be convenient if the second ruler had also another groove *r s*, so formed that when the shoulders *r* and *s* are in contact with the

incomparably more easy, and, consequently, more accurate in its application; insomuch, that there is no difficulty in giving an almost perfectly correct representation of any object adapted to this perspective, to which the artist has access, if he has a very simple knowledge of its principles, and a little practice.

It further represents the straight lines which lie in the three principal directions, all on the same scale. The right angles contained by such lines are always represented either by angles of 60 degrees, or the supplement of 60 degrees. And this, though it might look like an objection, will appear to be none on the first sight of a drawing on these principles, by any person who has ever looked at a picture. For, he cannot for a moment have a doubt, that the angle represented is a right angle, on inspection.

edges of the first ruler, the second should be at right angles to it. For representing circles in their proper positions, the writer made use of the inner edge of rims cut out from cards, into isometrical ellipses as represented in the figure; of these he had a series of different sizes, corresponding to his wheels. Such a series might be cut by help of the concentric ellipses in fig. 5, but he thinks that it would be an easier way to make use of that set of concentric ellipses as they stand, by putting them in the proper place under the picture, if the paper on which the drawing is made, be thin enough for the lines to be traced through, as by the help of them the several concentric circles will go to the representation of one which might be drawn at once. It is difficult to execute them separately with sufficient accuracy to make them correspond. For this purpose a separate plate of fig. 5 should be had, and one edge of the paper on the drawing-table should be loose to admit of the concentric ellipses being slid under it to the proper place, as described p. 215.

And we may observe further, that an angle of 60 degrees is the easiest to draw of any angle in nature. It may be instantly found by any person who has a pair of compasses, and understands the first proposition of Euclid. The representation, also, of circles and wheels, and of the manner in which they act on one another is very simple and intelligible. The principles of this perspective which, from the peculiar circumstance of its exhibiting the lines in the three principal dimensions on the same scale, I denominate "*Isometrical,*" will be understood from the following detail:

Suppose a cube to be the object to be represented. The eye placed in the diagonal of the cube produced. The paper, on which the drawing is to be made to be perpendicular to that diagonal, between the eye and the object, at a due proportional distance from each, according to the scale required. Let the distance of the eye, and consequently that of the paper, be indefinitely increased, so that the size of the object may be inconsiderable in respect of it.

It is manifest, that all the lines drawn from any points of the object to the eye may be considered as perpendicular to the picture, which becomes, therefore, a species of orthographic projection. It is manifest, the projection will have for its outline an equiangular and equilateral hexagon, with two vertical sides, and an angle at the top and bottom. The other three lines will be radii drawn from the centre to the lowest angle, and to the two alternate angles; and all these lines and sides will be equal to each

other both in the object and representation : and if any other lines parallel to any of the three radii should exist in the object, and be represented in the picture, their representations will bear to one another, and to the rest of the sides of the cube, the same proportion which the lines represented bear to one another in the object.

If any one of them, therefore, be so taken as to bear any required proportion to *its* object, *e.g.* 1 to 8, as in my representations of my models, the others also will bear the same proportion to *their* objects ; that is, the lines parallel to the three radii will be reduced to a scale.

I omit the demonstration of this, and some other points, partly for the sake of brevity, and partly because a geometrician will find no difficulty in demonstrating them himself, from the nature of orthographic projection ; and a person, who is not a geometrician, would have no interest in reading a demonstration.

For the same reason, it is unnecessary to show that the three angles at the centre are equal to one another, and each equal to 120 degrees, twice the angle of an equilateral triangle ; and the angle contained between any radius and side is 60 degrees, the supplement of the above, and equal to the angle of an equilateral triangle. All this follows immediately from Euclid, B. IV. Prop. 15, on the inscription of a hexagon in a circle.

In models, and machines, most of the lines are actually in the three directions parallel to the sides of a cube, properly placed on the object. And the

eye of the artist should be supposed to be placed at an indefinite distance, as before explained, in a diagonal of the cube produced.

Definitions.

The last mentioned line may be called the *line of sight.*

Let a certain point be assumed in the object, as for example c, fig. 2, Pl. I. and be represented in the picture, to be called, the *regulating point.* Through that point on the picture may be drawn a vertical line, c E, fig. 2, and two others, c B, c G, containing with it, and with one another, angles of 120°, to be called the *isometrical lines,* to be distinguished from one another by the names of the *vertical,* the *dexter,* and the *sinister* lines. And the two latter may be called by a common name—the *horizontal isometrical* lines. Any other lines, parallel to them, may be called respectively by the same names. The plane passing through the dexter, and vertical lines, may be called the *dexter isometrical plane;* that passing through the vertical, and sinister lines, the *sinister plane;* and that through the dexter and sinister lines, the *horizontal plane.*

By the use of the simple apparatus described above in the note, the representation of these lines in the objects may be drawn on the picture, and measured to a scale, with the utmost facility, the point at the extremity being first found, or assumed. The position of any point in the picture may be easily found, by measuring its three distances,

namely, first its perpendicular distance from the *regulating horizontal plane* (that is, the horizontal plane passing through the regulating point,) secondly, the perpendicular distance of that point where the perpendicular meets the horizontal plane, from the regulating dexter line; and thirdly, of the point, where that perpendicular meets the dexter line from the regulating point; and then taking those distances reduced to the scale, first, along the dexter line, secondly, along the sinister line, and thirdly, along the vertical line, in the picture. These three may be called the *dexter distance* of the point, its *sinister distance*, and its *altitude*. And it is manifest they need not be taken in this order, but in any other that may be more convenient to the artist, there being six ways in which this operation may be varied.

If any point in the same isometrical plane, with the point required to be found, is already represented in the picture, that point may be assumed as a new regulating point, and the point required found by taking two distances; and if the new assumed regulating point is in the same isometrical line with the point, it is found by taking only one distance. And this last simple operation will be found in practice all that is necessary for the determination of most of the points required. Thus any parallelopided, or any frame work, or other object with rafters, or lines lying in the isometrical directions, may be most easily and accurately exhibited on any scale required. But if it be necessary to represent lines in other directions, they will not be on the same scale,

but may be exhibited, if straight lines, by finding the extremities as above, and drawing the line from one to the other; or sometimes more readily in practice by help of an ellipse, as hereafter described.

If a curved line be required, several points may be found sufficient to guide the artist to that degree of exactness which is required.

The method of exhibiting the representations of any machines, or objects, the lines of which lie, as they generally do, in the isometrical directions; that is, parallel to the three directions of the lines of the cube, is as has been already shown; and likewise the mode of representing any other straight lines, by finding their extremities; or curved lines, by finding a number of points.

But in representing machines and models, there are not only isometrical lines, but also many wheels working into each other, to be represented. These, for the most part, lie in the isometrical planes. And it is fortunate that the picture of a circle in any one of these planes is always an ellipse of the same form, whether the plane be horizontal, dexter, or sinister; yet they are easily distinguished from each other by the position in which they are placed on their axle, which is an isometrical line, always coinciding with the minor axis of the ellipse.

This will be obvious from considering the picture of a cube with a circle inscribed in each of its planes, fig. 3, and considering these circles as wheels on an axle. The two other lines (or spokes of the wheel) in the ellipse, which are drawn respectively through the opposite points of contact of the circle with the

circumscribing figure, are isometrical lines also; for the points of contact bisect the sides of the circumscribing parallelogram, and therefore the lines are parallel to the other sides. They give likewise the true diameter of the wheels, reduced to the scale required. It further appears from the nature of orthographic projection, that the major axis of the ellipse is to the minor axis, as the longer to the shorter diagonal of the circumscribing parallelogram, that is (since the shorter diagonal divides it into two equilateral triangles,) as the square root of three to one; as appears from Euclid, Lib. I. Prop. 47: and since the sum of the squares of the conjugate diameters in an ellipse is always the same, if we put $\sqrt{1}$ for the minor axis, the $\sqrt{3}$ for the major, and i for the isometrical diameter, we shall have $2\,i^2 = 1 + 3, = 4$, and $i = \sqrt{2}$.

Therefore the minor axis, the isometrical diameter, and the major axis, may be represented respectively by $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$, or nearly by 1, 1·4142, 1·7321; or more simply, though not so nearly, by 28, 40, 49.

These lines may be geometrically exhibited by the following construction:

Let A B, fig. 4, be equal to B D, and the angle at B, a right angle. In B A produced, take B α = to A D draw α D, and produce both it, and α B. Then will B D, B α, and α D, be respectively to one another, as $\sqrt{1}$, $\sqrt{2}$, $\sqrt{3}$ by Euclid I. 47. Therefore if $\alpha\ \beta$ be taken equal to the isometrical diameter of the ellipse required, $\beta\ \delta$ drawn perpendicular to it will be the minor axis, and $\alpha\ \delta$ the major axis. The

ellipse itself, therefore, may be drawn by an elliptic compass, as that instrument may be properly set, if the major and minor axes are known. If it is to represent a wheel on an axle, care must be taken to make the minor axis lie along that axle. In the absence of the instrument it may be drawn from the concentric ellipses, fig. 5, which may be placed under the paper, in the position above described, and seen through it; if the paper be not too thick, and in this method the smaller concentric circles of the wheel may be described at the same time, as they may be seen through the paper, or if they should not be exactly of the right size, it would be easy to describe them by hand between the two nearest concentric ellipses; and thus also the height of the cogs of a wheel in the different parts of it may be exhibited longer and narrower towards the extremities of the minor axis. Their width may be determined from the divisions of the ellipse. In most cases this may be done with sufficient accuracy from the circumference of the ellipse being divided into eight equal divisions of the circle, by the two axes, and two isometrical diameters, each of which parts may be sub-divided by the skill of the artist; and not only the face of the wheel in front may be thus exhibited, but the parts of the back circles also, which are in sight, may be exhibited by pushing back the system of concentric ellipses on the minor axis or axle through a distance representing the breadth of the wheel, and then tracing both the exterior and the interior circles of the wheel, and of the bush on which it is fixed, as far as they are visible. Care

should be taken to represent the top of the teeth, or cogs, by isometrical lines, parallel to the axle, in a face-wheel, or tending to a proper point in the axle in a bevil-wheel. And nearly in the same way may the floats of a water-wheel be correctly represented. If a series of concentric ellipses, such as are given, fig. 5, be not at hand, it will still be easy for an artist to draw the ellipses with sufficient accuracy for most purposes, by drawing through the proper point in the axle, the major, and minor axes, and the two isometrical diameters, thus making eight points in the circumference to guide him.

If in any case it should become necessary to represent a circle, which does not lie in an isometrical plane, we may observe that the major axis will be the same in whatever plane it lies: and it will be the picture of that diameter, which is the intersection of the circle with the plane parallel to the picture, passing through its centre. And the major axis will bear to the minor axis the proportion of radius to the sine of the inclination of the line of sight to the plane of the circle. We may observe further, that the diameters of the ellipse, which are to the major axis, as $\sqrt{2}$ to $\sqrt{3}$, when such exist, are isometrical lines.*

And the representation of every other line parallel, and equal to any diameter of the circle, may be exhibited by drawing it equal and parallel to the

* We may remark, that if a cone be described, having its vertex at C which lies in the line of sight, fig. 2, and passing through the three radii C B, C E, C G, all the straight lines in the superficies of that cone passing through C, and all other lines parallel to any of them, are iso-

corresponding diameter in the ellipse. If it should be desired to divide the circumference of an ellipse into degrees, or any number of parts representing given divisions of the circle, it may be done by the following method :

Let an ellipse be drawn, fig. 6, and on its major axis, A G, a circle described, with its circumference divided into degrees or parts in any desired proportion, at B, C, D, E, F, &c. from which points draw perpendiculars to the major axis. They will cut the periphery of the ellipse in corresponding points. It would be difficult, however, in this way, to mark, with sufficient accuracy, the degrees, which lie near the extremities of the major axis. But the defect may be supplied by transferring those degrees in a similar way, from a graduated circle, described on the minor axis. In this manner an isometrical ellipse may be formed into an isometrical circular instrument, or an isometrical compass, which may show bearings or measure angles on the picture, in the same manner as a real compass or circular instrument would do in nature.

metrical, as well as those parallel to the three principal isometrical lines, C B, C E, C G ; and no other lines but these can be on the same scale. But though these multiply the number of isometrical lines infinitely, it is of little practical use, because it is only those which are parallel to the three principal lines, that can be easily distinguished at sight, to be isometrical.

We may further remark, that if a line be drawn through the point C parallel to any given line whatever, and that line be made to revolve round the line of sight, at the same angular distance from it, so as to describe the surface of a cone, all other lines parallel to it, in any of its positions, will be isometrical, as they respect one another.

It may be often useful to have a scale to measure distances, not only in the isometrical directions, but in others also. And this may be done by a series of similar concentric ellipses, as in fig. 7, dividing the isometrical diameters into equal portions. The other diameters will be so divided as to serve for a scale for all lines parallel to them respectively.

Thus, in the isometrical squares, exhibited in fig. 2, distances measured on the longer diagonal, or its parallels, would be measured by the divisions on the major axis, those depending on the shorter diagonal by the divisions on the minor axis.

To describe a cylinder lying in an isometrical direction, the circles at its extremities should be represented by the proper isometrical ellipses, and two lines touching both should be drawn: and in a similar way, a cone, or frustum of a cone, may be described. A globe is represented by a circle, whose radius is the semi-major axis of the ellipse representing a great circle.

It would not be difficult to devise rules for the representation of many other forms which might occur in objects to be represented. But the above cases are sufficient to include almost every thing which occurs in the representation of models, of machines, of philosophical instruments, and, indeed, of almost any regular production of art.

Buildings may be exhihited by this perspective as correctly, in point of measurement, as by plans and elevations, under the advantage of having the full effect of a picture.

A bridge, or any circular, or gothic arch, consisting of portions of circles lying in isometrical planes, may be represented by portions of isometrical ellipses, which will easily be adapted and drawn upon the principles already explained, by which wheels are exhibited on their axles. The centres of those circles must be found with which the centres of the ellipses must be made to coincide, their minor axes lying along the lines drawn from those centres perpendicular to the planes of the circles. The shaft of a pillar consists of a frustum of a cone and a cylinder united; or perhaps of a cylinder alone, or a congeries of cylinders: and we have already shown the method of exhibiting these, as well as their bases. And on the same principles, the position and size of the volutes and ornaments of the capital may be found, and such guiding points as will make it easy to trace their forms. Thus the different courts and edifices of a cathedral, a college, or a palace, may be correctly depicted; and even the rooms and internal structure, though less in the form of a picture, may be exhibited in such a way as to enable an architect, or his employer, to contemplate their situation, their ornaments, furniture, or any other circumstance belonging to their appearance, and to mark down exactly what he would have done, in such a way as could hardly be misunderstood by an attentive agent, though at a distance.

But in thus exhibiting buildings as transparent, and their interior laid open, there is a danger of being confused by a multiplicity of lines, which is a difficulty in a building containing many rooms, that

would need some address to get over. It is better adapted to exhibit the inside of a single room; of a cathedral, for instance, the aisles and transepts of which would not cause any great perplexity.

In the same manner a plan of a city might be given, which would not only represent its streets and squares, as well (by the help of the scale above described fig. 7,) as a common plan, but also a picture of its churches and public buildings, and even its private houses, if such were the design contemplated by the artist, as they would almost all become visible when looked down upon from the commanding height which this perspective supposes. And such a single exhibition, if well executed, might give a better idea of a distant capital than a volume of description.

In the instances which have been given, most of the lines are isometrical. But the art is applicable to many cases, where there are few, or none such. It may be necessary, in many of them, to draw isometrical lines, or isometrical ellipses, by way of a guide, to determine the position of certain lines and points to enable the artist to describe with accuracy what he has in view. And there is scarce any form so anomalous as to preclude the artist from taking advantage of these methods of ascertaining such lines or points in it as will give him much assistance in representing it with precision. If the intention be merely to make a picture, the guiding lines may be obliterated as soon as they have served the purpose designed, or they may be retained in some cases, and their lengths or diameters noted down in

figures, if it be wished, to give ready information. And often, if the artist wishes to provide materials to enable him at his leisure to give accurate descriptions or exact drawings, the rudest exhibition of such lines may completely serve his purpose, provided he notes down on the spot such measurements with accuracy, however unexact the lines may be on which they are recorded. In many cases it may be expedient to take liberties with this perspective, or with the picture, which will make it suit the purpose designed. And this will produce no confusion provided those liberties are explained: for instance, it may often be expedient to make the scale in the vertical direction larger, sometimes very considerably so, than in the horizontal. It may in some cases be necessary to represent on paper what is hid in nature. What has been said on the internal structure of buildings is an instance of this as well as what we shall observe on the exhibition of subterraneous objects. We shall proceed to give some examples of these observations.

To give such a representation of an Etruscan vase, as would enable an artist to model it exactly, would be exceedingly easy. Let a vertical line be drawn to represent the axis of the vase, fig. 8, and let points be taken in that axis, corresponding to the centres of the principal circles of the vase; through which the horizontal isometrical lines may de drawn representing the radii of those circles, by the help of which the isometrical ellipses representing them are easily drawn. These will become a complete guide to the artist. He may assist himself by looking at

the object along the line of sight, and then, if he has any skill in drawing, he will find no difficulty in tracing the outline from one of these to the other, with sufficient correctness. If he is unskilled in the art, of course he must be at the trouble of finding a larger number of ellipses to guide him. And in a similar manner, any solid formed by the revolution of a plane figure round one of its sides may be represented.

The laying down the timbers of a ship, or making a picture of one, shall be another example.

Let a vertical isometrical plane be conceived to pass through its keel, and to be intersected by the perpendicular planes passing through the ribs, and by planes parallel to the decks. The isometrical lines, which are the intersections of these, may be measured in the ship, and represented with their proper measures noted down in the picture, which will afford the means of representing the ribs, and laying them down in their proper places.

If this should be designed for the purpose of constructing a ship from a given model, it might be sufficient to represent the ribs only on one side; those on the other side being the exact counterparts. If the purpose should be to make use of these lines for a drawing, they need be marked but very faintly, and the artist will have little difficulty when guided by them to fill up the representation by hand.

A regular fortification, which we will suppose to have eight bastions, will afford another example.

A person not conversant in such a subject, is in

general puzzled with plans and sections, and has very little idea of what is meant to be conveyed.

But he would easily understand it if he should see every thing exhibited in a correct picture, especially where he has the view of his object varied, as in a fortification, such as has been proposed. Let an isometrical ellipse be drawn expressing the internal circumference of the place; and another concentric one, which marks the salient angles of the fortification on the principles already explained. Draw other guiding lines to every necessary point; the lines of the fortification may be easily transferred from a common plan to the isometrical by the help of the scale of concentric ellipses described above, fig. 7, which will serve also to lay down the length of the bastions and curtains, &c. in whatever direction they lie. Find the elevations of every part on the isometrical scale; and thus the body of the place, the ditches, counterscarp, covered way, glacis, ravellins, and all the outworks, will be represented to the eye as they appear in reality, and in every varied position, with the advantage of having all the admeasurements laid down with geometrical precision.

If the artist should think the vertical lines in such an exhibition too small to give a correct idea of all the minute elevations, there would be no harm in his increasing the scale in that dimension in any desired proportions.

The face of a hilly or mountainous country like Switzerland, or the district of the lakes in the north part of England, will afford another example.

Isometrical horizontal lines may be drawn representing lines in the level from which the height of the mountains is to be reckoned, so that vertical lines drawn from the summits of the mountains may meet them, on which the heights may be marked; (as well as recorded in figures, if required.) And the mountains themselves may be drawn in their topographical situation. Their bearings may be marked by the help of the isometrical compass described in page 219. It would be easy to transfer them from a common map to the isometrical plan; and thus the face of the country might be represented just as it would appear from the commanding height which the isometrical perspective supposes.

Yet, as the slopes of hills and mountains are seldom so steep as the line of sight, it might sometimes suit the purpose to represent the height of elevations as twice or three times the reality, in order that mountains might project an outline on the plane behind; otherwise, the summit might be projected on the mountain itself, which would, in a degree, destroy the effect of a picture.

This art might be advantageously employed also for tracing what is below the surface of the earth, as well as what is above it. It may be applied to geological purposes, and give, not only the order of the strata, but their variations and their geographical situations. And for this purpose it might be useful to increase the vertical scale, in a great proportion, above the horizontal. It would be easy to mark the dip, or rise of the strata, as well as of the earth above them: to represent their various disruptions,

to show the situation and extent of fissures and metallic veins, to mark the boundaries where the upper strata have been swallowed up, or cease to appear, or where the under strata push up towards the day. It would be easy to mark the variations in the thickness of the strata in different places, and to record the result of experiments made at any point, by boring or sinking shafts, which might be done by drawing a vertical line downward, so as to represent the thickness of the laminæ, which might be marked by different colours. By such a method, the geologist might obtain a map of the country, which might exhibit, at one view, the general results of all the experiments and inquiries that had been made relative to that science. And the owner of an estate might record in a small compass, all that is known respecting its minerals, and be able from a comprehensive view of them all, to judge of the probability of success in sinking a shaft, or driving a level. He might also make good use of this perspective in tracing his shafts and drifts in all their windings, elevations, and depressions, and comparing them with the surface above, marking also the veins and strata in which they run. For if the artist knows what is beneath the surface, he has no difficulty in representing it as transparent. He must be careful however not to perplex himself by lines too mueh multiplied, and take advantage of his being able to paint the lines with different colours, for the purposes of distinction ; and he must use a considerable address in throwing out such lines as would

be of little use, and retaining such as will produce the effect of a picture, which should be well preserved in order to make the exhibition easily intelligible.

If he should wish to make a drawing of minerals or crystals, this perspective would be well suited to the purpose.

The point, however, on which the writer of this paper can speak with the greatest confidence is on the representation of machines and philosophical instruments ; having been himself so much in the habit of practically applying to them the principles that have been detailed : and this he has exemplified in the plates.

The correct exhibition of objects would be much facilitated by the use of this perspective, even in the hands of a person who is but little acquainted with the art of drawing ; and the information given by such drawings is much more definite and precise than that obtained by the usual methods, and better fitted to direct a workman in execution.*

* The author has transcribed this interesting paper from the first volume of the Transactions of the Cambridge Philosophical Society. The method is peculiarly deserving of the attention of mechanics and engineers.

THE END.